LimitState:RING Manual

VERSION 3.2.b

LimitState Ltd

November 10, 2016

LimitState Ltd
The Innovation Centre
217 Portobello
Sheffield S1 4DP
United Kingdom
T: +44 (0) 114 224 2240
E: info@limitstate.com
W: http://www.limitstate.com

© LimitState Ltd

LimitState:RING

© LimitState Ltd

All rights reserved. No parts of this work may be reproduced in any form without the written permission of LimitState Ltd.

While every precaution has been taken in the preparation of this document, LimitState Ltd assumes no responsibility for errors or omissions. LimitState Ltd will not be liable for any loss or damage of any kind, including, without limitation, indirect or consequential loss (including loss of profits) arising out of the use of or inability to use this document and/or accompanying software for any reason.
This document is provided as a guide to the use of the software. It is not a substitute for standard references or engineering knowledge. The user is assumed to be conversant with standard engineering terminology and codes of practice. It is the responsibility of the user to validate the software for the applications for which it is to be used.

© LimitState Ltd

Contents

I Introduction and Quickstart **13**

1 Introduction **15**
1.1 About LimitState:RING . . . 15
1.2 Glossary . . . 15
1.3 LimitState:RING terminology . . . 16
1.4 About LimitState . . . 17
1.5 Using Help . . . 17
1.6 System requirements . . . 17
1.7 Program limits . . . 18
1.8 Contact details . . . 18
1.8.1 Sales . . . 18
1.8.2 Software support . . . 18
1.8.3 Website . . . 18

2 What's new in LimitState:RING **19**
2.1 More flexible arch profile definition . . . 19
2.2 Moment, normal and shear force diagrams . . . 19
2.3 Upgraded solver engine . . . 19
2.4 Reinforced masonry . . . 20
2.5 Expanded vehicle database . . . 20
2.6 Enhanced user interface . . . 20

3 Getting started **21**
3.1 Installation & licensing . . . 21
3.2 Starting LimitState:RING . . . 21

4 Quick Start Tutorial **23**
4.1 Using the 'New bridge wizard' . . . 23
4.1.1 Introduction to the wizard . . . 23
4.1.2 Step 1 - General project settings . . . 24
4.1.3 Step 2 - Geometry . . . 24
4.1.4 Step 3 - Partial Factors . . . 25
4.1.5 Step 4 - Materials . . . 26
4.1.6 Step 5 - Add vehicles . . . 26
4.1.7 Step 6 - Choose mode of operation . . . 27
4.2 Following an analysis . . . 28
4.3 Output from analysis . . . 29
4.4 Modifying properties with the property editor . . . 29

II Theory **31**

5 Theoretical basis of LimitState:RING **33**
5.1 Background . . . 33
5.2 Analysis methods used by LimitState:RING . . . 34
5.3 Output from a LimitState:RING analysis . . . 37
5.3.1 Identification of the adequacy factor . . . 37
5.3.2 Bridge behaviour when subjected to support movements . . . 37
5.4 Range of applicability of LimitState:RING . . . 39
5.4.1 Spans . . . 39
5.4.2 Block shape . . . 39
5.4.3 Stress-related failures . . . 39
5.4.4 Fill depth . . . 40
5.4.5 3D effects . . . 40
5.4.6 Range of collapse modes identifiable . . . 41
5.5 Use of reinforcement . . . 41
5.6 Finite masonry strength . . . 43
5.7 Sliding failures . . . 44
5.8 Backfill . . . 45
5.8.1 General . . . 45
5.8.2 Dispersion of live loads . . . 45
5.8.3 Passive restraint . . . 46
5.8.4 Backing . . . 48

III Modelling **49**

6 Preliminary bridge assessments using LimitState:RING **51**

7 Detailed bridge assessments using LimitState:RING **53**
7.1 Analysis parameters . . . 53
7.1.1 Partial factors of safety . . . 53
7.2 Modelling the shape of the arch . . . 54
7.3 Skew bridges . . . 55
7.4 Multi-span bridges . . . 55
7.5 End abutment blocks . . . 56
7.6 The influence of infill material . . . 56
7.7 Modelling the mechanical properties of masonry . . . 57
7.8 Modelling bridge defects . . . 58
7.8.1 Missing mortar and/or localised spalling of masonry units . . . 58
7.8.2 Ring separation in multi-ring brickwork arches . . . 59
7.8.3 Cracking in the arch barrel . . . 61
7.9 Modelling flooded masonry arch bridges . . . 63

8 Loading models **65**
8.1 Loading from railway vehicles . . . 65
8.1.1 Railway loading models . . . 65
8.1.2 Distribution of rail loads through the track . . . 65
8.1.3 Longitudinal distribution of load through ballast and fill . . . 66
8.1.4 Transverse distribution and effective bridge width . . . 67
8.1.5 Dynamic effects . . . 67
8.1.6 Other effects . . . 69

8.2 Loading from highway vehicles . . . 69
8.2.1 Highway loading models . . . 69
8.2.2 Transverse distribution and effective bridge width . . . 70
8.2.3 Dynamic effects . . . 71
8.2.4 Other effects . . . 71

9 Using other LimitState:RING features to investigate bridge behaviour 73
9.1 Identifying the causes of observed cracks using the 'Support Movement' feature 73
9.2 Exploring load paths under service loads . . . 73
9.3 Modelling bridge spans with intermediate supports . . . 76

10 Interpreting output from LimitState:RING 77
10.1 Adequacy factor . . . 77
10.2 Mode of response . . . 78
10.3 Zone of thrust / internal forces . . . 79

IV User Guide 81

11 The Graphical Interface 83
11.1 Introduction . . . 83
11.2 Title Bar . . . 84
11.3 Menu Bars . . . 84
11.4 Toolbars . . . 84
11.5 Viewer Pane . . . 85
11.6 Property Editor . . . 86
11.7 Output Pane . . . 87
11.8 Status Bar . . . 87
11.9 Rearranging the toolbars . . . 87
11.10 Detaching a menu . . . 88
11.11 Calculator . . . 88

12 Project details 89
12.1 Required details . . . 89
12.1.1 Bridge type . . . 90
12.1.2 Effective bridge width . . . 90
12.1.3 Model reinforcement . . . 91
12.2 Optional details . . . 91

13 Bridge geometry 93
13.1 Geometry dialog . . . 93
13.2 Abutments . . . 94
13.2.1 Default abutment model . . . 94
13.2.2 Modelling abutments explicitly . . . 94
13.3 Spans . . . 95
13.3.1 Number of rings . . . 95
13.3.2 Arch type . . . 95
13.3.3 Number of units . . . 96
13.3.4 Ring thickness . . . 96
13.3.5 Inserting a span . . . 97
13.3.6 Deleting a span . . . 97
13.4 Piers . . . 97

13.4.1 Default pier model . . . 98
13.4.2 Modelling piers explicitly . . . 98
13.5 Fill profile . . . 98

14 Material properties **101**
14.1 Masonry . . . 101
14.1.1 'Specify properties for' . . . 101
14.1.2 Unit weight . . . 102
14.1.3 Model crushing . . . 102
14.1.4 Sliding properties . . . 103
14.2 Backfill - Standard properties . . . 104
14.2.1 Soil Properties . . . 104
14.2.2 Soil Effects . . . 105
14.3 Backfill - Advanced properties . . . 105
14.3.1 Live load dispersion details . . . 105
14.3.2 Soil-arch interface properties . . . 107
14.3.3 Passive zone parameters . . . 107
14.4 Backing properties . . . 110
14.5 Surface fill . . . 110
14.5.1 Unit weight . . . 110
14.5.2 Angle of dispersion . . . 111

15 Loading **113**
15.1 Mirrored vehicles . . . 113
15.2 Adding a vehicle to the project . . . 114
15.2.1 Importing existing vehicles . . . 114
15.2.2 Defining a new vehicle using properties saved in a file . . . 115
15.2.3 Defining a new vehicle within LimitState:RING . . . 116
15.2.4 Editing vehicle properties . . . 116
15.2.5 Renaming a vehicle . . . 117
15.2.6 Deleting a vehicle . . . 117
15.2.7 Exporting a vehicle to a file . . . 117
15.3 Adding a vehicle to a load case . . . 117
15.3.1 Assigning 'dynamic' factors . . . 118
15.4 Defining new load cases . . . 118
15.4.1 Adding load cases . . . 118
15.4.2 Deleting a load case . . . 119
15.5 Moving vehicles across a bridge . . . 120
15.5.1 Method 1 - Multiple load cases . . . 120
15.5.2 Method 2 - Drag and solve . . . 121
15.6 Moving a load in a multiple load case problem . . . 121
15.7 Viewing load cases . . . 121

16 Partial factors **123**

17 Reinforcement **125**
17.1 Properties . . . 125
17.2 Adding reinforcement to the project . . . 126

18 Support movement analysis **129**
18.1 Background . . . 129
18.2 Support movement wizard . . . 130

19 Viewing and modifying attributes **131**
19.1 Using the property editor . . . 131
19.1.1 Fill element(s) . . . 132
19.1.2 Block(s) . . . 133
19.1.3 Contact(s) . . . 134
19.2 Using the explorers . . . 135
19.2.1 Opening an explorer . . . 135
19.2.2 Navigating the explorers . . . 136
19.2.3 Editing data . . . 136

20 Display options **139**
20.1 General . . . 139
20.1.1 Language specific variations . . . 139
20.1.2 Scrollbars . . . 139
20.1.3 Current mouse position . . . 139
20.1.4 Scrolling wheels . . . 139
20.2 Viewer pane . . . 140
20.2.1 Rotating the model . . . 140
20.3 Menus . . . 142
20.3.1 File menu . . . 142
20.3.2 Edit menu . . . 143
20.3.3 Select menu . . . 143
20.3.4 View menu . . . 144
20.3.5 Tools menu . . . 145
20.3.6 Analysis menu . . . 145
20.3.7 Help menu . . . 146
20.4 Toolbars . . . 146
20.4.1 Default toolbars . . . 146
20.4.2 Optional toolbars . . . 147
20.5 Context menus . . . 147
20.5.1 Viewer pane context menu . . . 147
20.5.2 Toolbar / property editor context menu . . . 148
20.5.3 Explorer context menu . . . 149

21 Analysis **151**
21.1 Auto-solve . . . 151
21.2 Types of analysis . . . 151
21.2.1 Normal analysis . . . 151
21.2.2 Iterative analysis . . . 151
21.3 The solvers . . . 152
21.4 Analysis results . . . 153
21.4.1 Adequacy factor found . . . 153
21.4.2 No solution found . . . 153
21.4.3 Aborting an analysis . . . 153

22 Post analysis functions **155**
22.1 Visual output . . . 155
22.1.1 Force diagrams . . . 156
22.2 Quantitative output . . . 156

© *LimitState Ltd*

23 Report output **159**
23.1 Viewing report output . . . 159
23.2 Adding a template, header or footer . . . 160

V Appendices **163**

A Mathematical formulation **165**
A.1 Joint equilibrium formulation (adequacy factor analysis) . . . 165
A.2 Joint equilibrium formulation (support movement analysis) . . . 166
A.3 Including finite masonry material strength . . . 166
A.3.1 Algorithm . . . 167
A.4 Including reinforcement . . . 168
A.5 Worked example . . . 170
A.5.1 Equilibrium constraints . . . 170
A.5.2 Yield constraints . . . 172
A.5.3 Objective function . . . 172
A.5.4 Problem matrix . . . 172

B Additional notes on the backfill model **175**
B.1 Boussinesq distribution model . . . 175
B.2 Limiting horizontal fill stresses . . . 176
B.3 Passive and active fill pressures . . . 177
B.4 Gradual build-up of passive pressures . . . 179
B.5 Unusual failure mechanisms . . . 180
B.6 Backfill - Backwards compatibility with RING 1.5 . . . 181
B.6.1 Unit weight . . . 181
B.6.2 Limiting fill / barrel angle of friction . . . 181
B.6.3 Load dispersion type . . . 182
B.6.4 Horizontal pressure type . . . 182
B.6.5 Automatic identification of passive zones . . . 183

C Default parameters **185**
C.1 General . . . 185
C.2 Geometry . . . 185
C.3 Transverse properties (only used with auto-computed bridge width) . . . 186
C.4 Material properties . . . 187
C.5 Partial factors . . . 188
C.6 Track parameters . . . 188

D Standard loading models **189**
D.1 Railway loading models . . . 190
D.1.1 UIC . . . 190
D.1.2 BD37 . . . 191
D.1.3 Network Rail (NR/GN/CIV/025) . . . 191
D.1.4 Indian Railways . . . 193
D.2 Highway loading models . . . 194
D.2.1 Construction and use . . . 194
D.2.2 Restricted construction and use . . . 195
D.2.3 European Union vehicles . . . 196
D.2.4 BD21 (Annex A) / AW (Schedule 3) . . . 197
D.2.5 BD21 (Annex D) . . . 198

© *LimitState Ltd*

D.2.6 BD21 (Annex E) . . . 199
D.2.7 BD21 (Annex F) . . . 200
D.2.8 BD37 (HB Loading) . . . 201
D.2.9 BD86 Special Vehicles . . . 202
D.2.10 BD91 . . . 204

E Worked examples - general **205**
E.1 Example 1 - single span stone voussoir underline railway bridge . . . 205
E.1.1 Details . . . 205
E.1.2 Assessment data . . . 207
E.1.3 Analysis results . . . 208
E.1.4 Next steps . . . 209
E.2 Example 2 - multi-span, multi-ring brickwork underline railway bridge . . . 210
E.2.1 Details . . . 210
E.2.2 Assessment data . . . 211
E.2.3 Analysis results . . . 213
E.2.4 Next steps . . . 214

F Worked examples - reinforcement **217**
F.1 Case 1 - All reinforcement in full tension . . . 217
F.2 Case 2 - Bottom reinforcement in full tension, top reinforcement in partial tension 219
F.3 Case 3 - Bottom reinforcement in full tension, top reinforcement in full compression 221

G Validation against bridge test results **223**
G.1 Bolton laboratory tests (full-scale) . . . 223
G.2 Sheffield laboratory tests (small-scale) . . . 226
G.3 Salford laboratory tests (full-scale) . . . 227
G.4 Field bridge tests . . . 229
G.5 Validation of the reinforcement model . . . 230
G.5.1 Bradford arches . . . 230
G.5.2 TRL laboratory bridge tests . . . 230

H Comparison with previous versions **233**
H.1 Version history . . . 233
H.2 Comparison of results between versions . . . 233

Bibliography **237**

© *LimitState Ltd*

© LimitState Ltd

Part I

Introduction and Quickstart

Chapter 1

Introduction

1.1 About LimitState:RING

LimitState:RING is a rapid analysis tool for masonry arch bridges. The software is primarily designed to analyze the ultimate load carrying capacity of masonry arch bridges (building on the 'mechanism' method of analysis originally pioneered by Heyman 1982), and has numerous features, many of which are unique, including:

- Multi-span and multi ring arch capabilities
- Multiple load case facility
- Facility to model support movements
- Facility for modelling the presence of arch backing material
- Fully user-definable geometry
- Local material properties can be specified
- Effective width computations
- Automatic identification of the critical failure mode in multi-span bridges, even if this involves only a single span
- Failure modes involving sliding are identified if critical
- Automatic detection of 'passive' pressures, allowing deep arch and multi-span arch problems involving passive pressures to be analyzed without difficulty
- Original version validated by academia and industry (e.g. see Gilbert & Melbourne 1994, Melbourne & Gilbert 1995 and Melbourne et al. 1997); new features informed by ongoing active research.

1.2 Glossary

Masonry arch bridges are very different to the steel and concrete bridges which are instead constructed in their place today. As the terminology used to describe different parts of masonry arch bridges can appear obscure to the non-specialist, common terms are given in Figure 1.1.

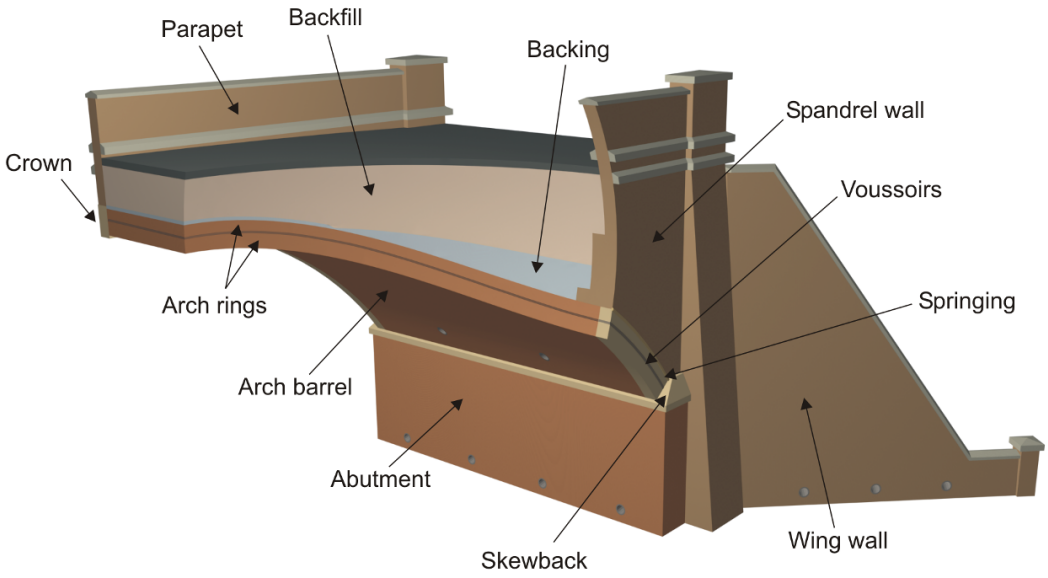


Figure 1.1: Masonry arch bridge terminology

1.3 LimitState:RING terminology

LimitState:RING idealizes a bridge as a series of *blocks* separated by *contacts* (where sliding / crushing / hinging can occur), with the effects of fill modelled by live load dispersal and restraint from backfill elements. The annotated image displayed in Figure 1.2 highlights the most important objects the user will encounter when using LimitState:RING.

Upon solving, LimitState:RING determines the critical failure mode, with *hinges* often forming as sections of the arch push against *backfill elements* (designed to replicate the effect of the passive restraint offered by the fill). Finally, the *thrust zone* at collapse is also shown. This gives a visual indication of both the position of the line of compressive force and the minimum amount of material needed to support it.

© *LimitState Ltd*

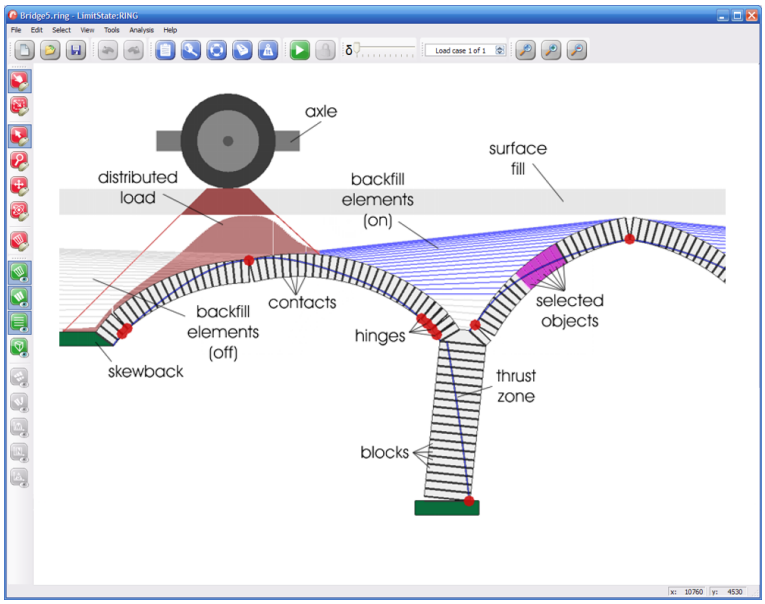


Figure 1.2: The main objects encountered in LimitState:RING

1.4 About LimitState

LimitState is a University of Sheffield spin-out company specializing in the development of powerful yet easy-to-use software applications which use unique technology to rapidly identify critical collapse mechanisms and associated margins of safety. This allows engineers to move beyond simple 'automated hand calculations' and predefined mechanisms - but without the need to resort to significantly more complex and potentially cumbersome techniques (e.g. non-linear finite element analysis).

LimitState co-founder Dr Matthew Gilbert is a Chartered Civil Engineer who has been involved in masonry arch bridge assessment and research since 1990, developing the first version of RING in 1992.

1.5 Using Help

The software includes an online help facility which is largely based on this Program Reference Manual. Pressing F1 at any time will activate the help system.

1.6 System requirements

LimitState:RING runs on Windows XP, Vista, 7, 8 and 10 Operating Systems (support for OSX and Linux is available on request, subject to demand). Recommended system specifications are as follows (ideal values given in parenthesis):

- 500MHz (1.5+ GHz) Intel (or compatible) processor
- 120Mb (250+ Mb) free hard disk space
- 512Mb (1+ Gb) RAM

1.7 Program limits

The program uses a 'Single Document Interface' which means that only one bridge project file can be open in LimitState:RING at a given time. However, several instances of LimitState:RING can be opened simultaneously if required and each of these may contain a separate bridge project file.

Previous versions of the software (e.g. RING 1.5) imposed limits on the number of rings, blocks etc. which could be modelled. In LimitState:RING 3.2.a problem size is limited only by available computer power.

1.8 Contact details

1.8.1 Sales

To request information on pricing, a formal quotation, or to purchase the software please contact LimitState Ltd, at **sales@limitstate.com**.

1.8.2 Software support

Software support for LimitState:RING is available to all users with valid maintenance contracts. All queries should be directed to **support@limitstate.com**.

1.8.3 Website

For the most up-to-date news about LimitState:RING, please visit the LimitState:RING website: www.limitstate.com/ring.

Chapter 2

What's new in LimitState:RING

2.1 More flexible arch profile definition

A number of new arch profile types have been added, potentially allowing users to more closely model the shape of a given arch bridge:

User-defined (interpolated) The arch profile is formed from a spline interpolation of user-defined data points. This is particularly suitable for use when many user-defined data points are specified.

Three-centered (pseudo-elliptic) The arch profile is (near) elliptical in shape, being formed from segments of three circles using the crown rise and span measurements.

Pointed The arch profile is pointed and is formed from segments

Find out more about arch profile definition in Section 13.3.

2.2 Moment, normal and shear force diagrams

Users can now gain an improved understanding of the way in which applied loads are resisted through the display of moment, normal and shear force diagrams. (These are particularly useful when reinforcement is involved - when plotting the zone of thrust is not particularly useful).

Find out more about moment and force diagrams in LimitState:RING in Chapter 22.

2.3 Upgraded solver engine

Designed to solve large-scale mathematical optimization problems in a highly efficient manner, the use of Mosek 6 linear optimizer in LimitState:RING means that larger, more complex, problems can be solved much more quickly than before.

2.4 Reinforced masonry

LimitState:RING allows the user to include reinforcement in the model, meaning that the the software can also be used to assess a range of reinforced arch bridges.

Find out more about adding reinforcement in LimitState:RING in Chapter 17.

2.5 Expanded vehicle database

The built-in database of road and rail vehicles has been expanded for LimitState:RING and now includes the following:

Railway vehicles:

UIC (LM71 and Load trains D4, C3, E4 and E5)

BD37 (RU and RL railway loading)

Network Rail (RA1 and RA10 standard and short length load trains)

Indian Railways (Modified broad gauge A and B trains)

Highway vehicles:

Construction and Use (Single, double and triple axle load vehicles)

Restricted Construction and Use (RA to RG load vehicles)

European Union (Single, double and triple axle vehicles plus EC1 to EC4 load vehicles)

BD21 (Load vehicles from Annexes A, D, E and F)

BD37 (1, 30, 37.5 and 45 Unit HB loading)

BD86 (SV80, SV100, SV150, SVTrain [SV196] and SVTT load vehicles)

BD91 (32 to 44 tonne load vehicles)

Find out more about the standard vehicles included with LimitState:RING in Appendix D.

2.6 Enhanced user interface

The look and feel of the user interface has been enhanced for LimitState:RING. Toolbar buttons are now colour-coded and rendering of bridges displayed in the viewer has been improved. Other improvements to the user experience, such as the ability to cut and paste span profile and surface fill level data to and from external applications (e.g. Microsoft Excel) have also been implemented.

© *LimitState Ltd*

Chapter 3

Getting started

3.1 Installation & licensing

Further details on installation and licensing are provided in the separate Installation manual distributed with the software.

3.2 Starting LimitState:RING

To start LimitState:RING, on the **Start** menu, point to **Programs** and click **LimitState:RING**. A few seconds after starting LimitState:RING the welcome screen shown in Figure 3.1 should appear.

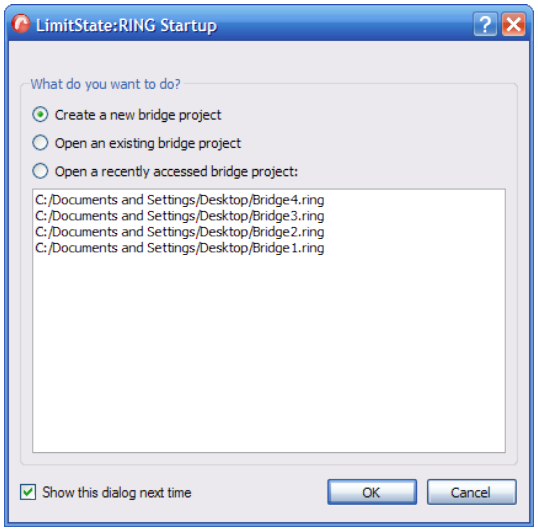


Figure 3.1: LimitState:RING welcome screen

You then have three options:

1. **Create a new bridge project** - select this option and click **OK** to bring up the Chapter 4.
2. **Open an existing bridge project** - select this option and click **OK** to open a previously saved project or example.
3. **Open a recently accessed bridge project** - select this option, choose a file from the list and click **OK** to return to a recent project.

© *LimitState Ltd*

Chapter 4

Quick Start Tutorial

The easiest way to get started using LimitState:RING is to select **Create a new bridge project** and click **OK**; the **New bridge wizard** will then start on the **Project Details** tab (Figure 4.1). Alternatively, click the **New** icon in the **File** toolbar, .

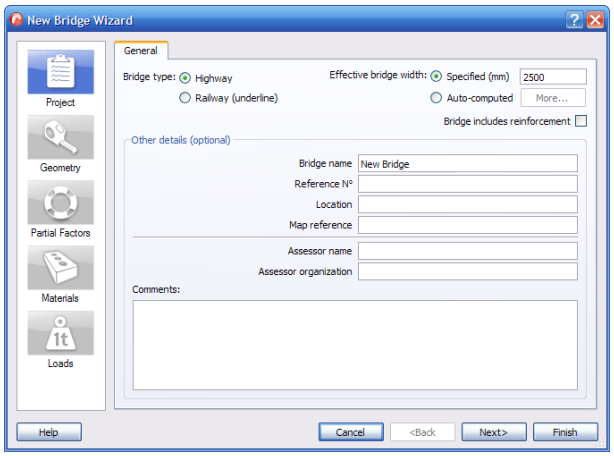


Figure 4.1: Wizard - Project Details

4.1 Using the 'New bridge wizard'

The **New bridge wizard** guides the user through the process of defining the bridge **Geometry** (see Section 4.1.3), **Materials** (see Section 4.1.5) and **Applied loading** (see Section 4.1.6). Further explanation of all parameters that can be modified is given elsewhere (all dialog pages displayed as part of the wizard are also accessible from the **Tools** menu, Section 20.3.5).

4.1.1 Introduction to the wizard

The **New bridge wizard** function in LimitState:RING is designed to help the user quickly generate a model of their structure.

It should be noted that, at any point during the Wizard process, it is possible to click **Finish**. LimitState:RING will automatically fill in any information that has not been explicitly supplied by assuming default values and using information already given up to that point.

In most cases, the Wizard process involves entering information in a sequential manner, clicking **Next**> after each step. However, it is possible to move backwards through the various steps by using the <**Back** button. The left-hand pane of the Wizard dialog serves as a reference point, with the current section being highlighted in blue.

4.1.2 Step 1 - General project settings

Many of the fields in the ***Project*** tab are optional. However, those at the top of the tab determine the fundamental type of model you will build and specify several other important parameters.

Firstly, specify the type of bridge to be analysed - this can be either *Highway* or *Railway (underline)*.

Next, if you wish to model a bridge that includes reinforcement, tick the ***Bridge includes reinforcement?*** box. Upon completing the wizard, you will be alerted to the fact that reinforcement properties can be specified using the **Contact Select Tool** and the **Property Editor**.

Lastly, enter an ***Effective bridge width***. This is the transverse width of masonry arch that resists the applied loading. Enter a constant value, using the *Specified* field or, if you wish to use an automatically calculated effective width, select the *Bridge Width: Auto-computed* option (see the **Loading** Chapter, Section 8).

4.1.3 Step 2 - Geometry

The next stage in the Wizard process is to model the **Geometry** (see Section 13) of the bridge (Figure 4.2). This includes data about the abutments, spans, piers and fill.

In the case of the abutments, there is an **Advanced** (see Section 13.2.2) button that will allow explicit modelling of the height, thickness and number of blocks. However, it is advised that abutments are only modelled explicitly in **special circumstances** (see Section 7.5).

Move through the tabs, filling in the relevant data and clicking **Next**> to advance.

On the **Span** tab, there is the option to insert additional spans into the model. If you wish to do this during the Wizard process, simply check the **Insert span after this?** box and, after clicking **Next**>, a new span and pier will be added to the right of the current position.

If a span has been accidentally omitted, but the Wizard process is not yet complete, additional spans can still be included. To do this, simply navigate back to a span adjacent to the position where the new span is required and click the **Insert Span** button. A dialog will appear to ask whether the new span should be positioned to the right or left of the current one.

Should a span require deleting, navigate to the correct tab of the Wizard and click the **Delete Span** button. A dialog will appear the asks which of the two supporting piers should also be removed.

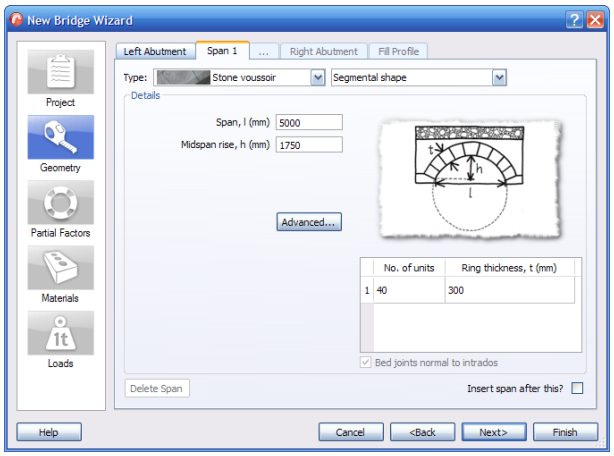


Figure 4.2: Wizard - Geometry

Note:

1. *It is not possible to delete the last remaining span or the abutments.*

2. *For more detailed information on editing the geometry of the bridge, see* Section 13.

4.1.4 Step 3 - Partial Factors

On the **Partial Factors** tab, insert the partial factors relevant to the code of practice being used (see Figure 4.3).

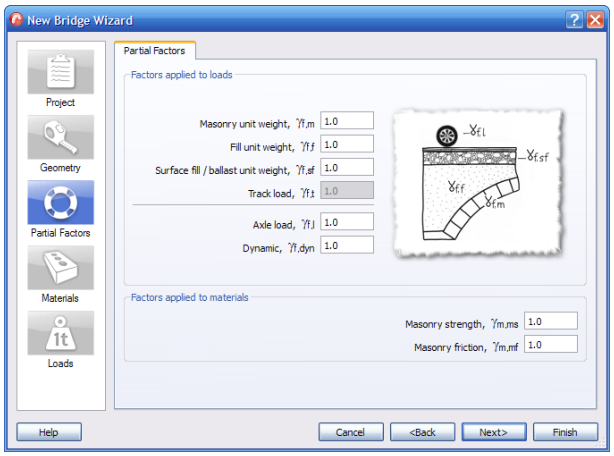


Figure 4.3: Wizard - Partial Factors

Note: For more detailed information on editing the partial factors, see Section 16.

4.1.5 Step 4 - Materials

On the **Masonry** tab (Figure 4.4), by using the **Specify properties for** drop-down menu, it is possible to specify:

1. The same properties for all the masonry in the bridge (*All masonry*).

2. Different sets of properties for the spans, skewbacks and piers / abutments (*Spans vs. piers/abutments*).

3. Different properties for each span, pier and abutment in the bridge (*All bridge parts*).

On the **Backfill** tab, clicking on the **Advanced...** button allows access to more specialized soil properties.

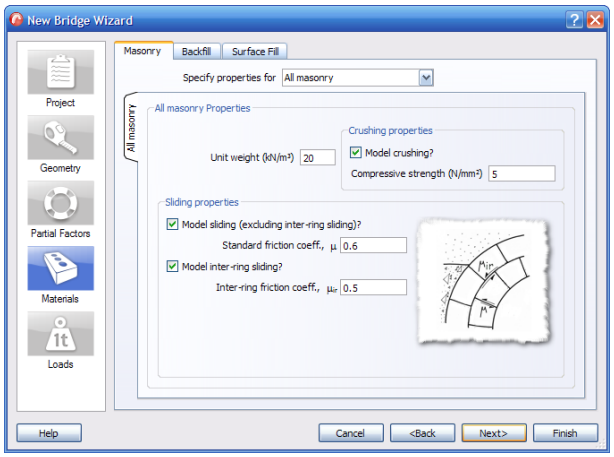


Figure 4.4: Wizard - Material Properties

Note:

1. *For more detailed information on editing the backfill properties, see* Section 14.2.

2. *For more detailed information on editing the bridge materials, see* Section 14.

4.1.6 Step 5 - Add vehicles

Using the **Vehicle database** (see Section 15.2), specify the vehicles to be used in the current project as shown in Figure 4.5:

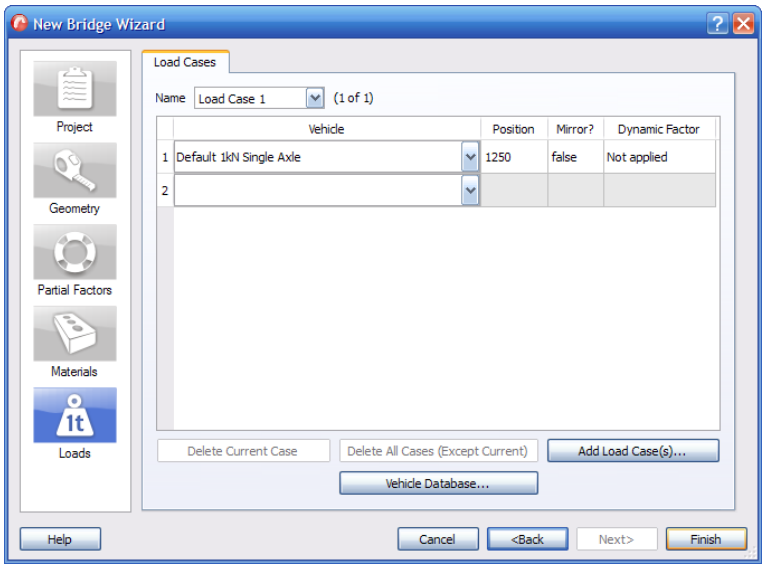


Figure 4.5: Wizard - Adding loads

1. Click on the **Vehicle Database** button. A dialog will appear that includes all of the pre-defined vehicle types in a library on the left.
2. Select the vehicle to be used from the library **tree** and click the **Import** button to move it across into the **Project vehicles** section.
3. After all the required vehicle types have been imported, click **OK** to return to the Load Case dialog.
4. Use the drop-down menus to select the required vehicle for the current load case.
5. Specify the position of the vehicle, whether it is mirrored and, if appropriate, assign axles to be subject to the ***dynamic*** partial factor of safety.

4.1.7 Step 6 - Choose mode of operation

At this point in the Wizard, the user should decide upon the way in which they will use LimitState:RING.

Drag and solve

If the user wishes to better understand the behaviour of their model, it is recommended that the **Drag and solve** (see Section 21.1) mode is used. To do this:

1. Complete the Wizard by clicking **Finish**.
2. Move the loading vehicle to the required position.
3. Click the **Solve** button .

If the user finds that they use this method for the majority of their problems, the **Auto-solve** feature can be enabled. This will cause LimitState:RING to automatically solve each time the vehicle is moved. To enable this feature, check the **Solve automatically...** box in the **Preferences** dialog (located in the **Tools** menu).

Multiple load cases

Alternately, a vehicle can be moved incrementally across a bridge by setting up a series of load cases. In this way the user can quickly identify the critical load position:

- Click the **Add Load Case** button.
- Check the option to **Copy an existing load case**.
- Specify the target load case (usually load case 1).
- Choose the number of times to copy the load case and the offset for each (*e.g.* to model a vehicle at 10 positions over a 3000mm bridge, copy 'load case 1' 9 times with a 300mm offset).
- Click **OK** to return to the **Loading** dialog.
- Complete the Wizard by clicking **Finish**.
- Click the **Solve** button .

Note: For more detailed information on editing the bridge loading, see Section 15.

4.2 Following an analysis

The collapse mechanism and associated adequacy factor (multiplier on factored vehicle loads) will be displayed after a short time (see Figure 4.6). The processing time depends on the complexity of the problem specified and on the speed of the PC being used.

The output window will display details of the analysis and it is also possible to view moment, shear and normal **force diagrams** (see Section 22.1.1) diagrams for the structure.

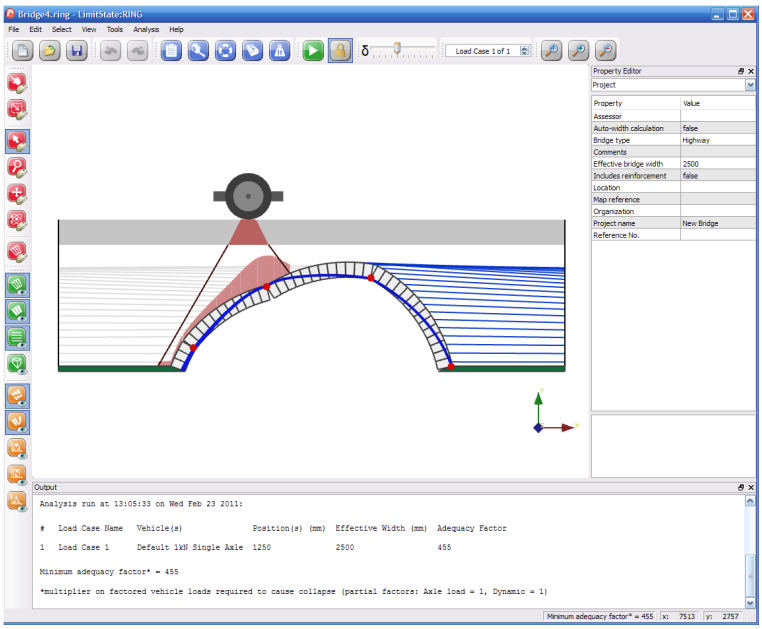


Figure 4.6: Collapse mechanism and adequacy factor following a LimitState:RING analysis

Note: For more detailed information on performing an analysis, refer to Section 21.

4.3 Output from analysis

On the **Analysis menu** (see Section 20.3.6) click **Report...**. This command opens a window displaying full details of the model and analysis results. It is then possible to edit this file (if necessary) and then to output to a printer or pdf file for later retrieval.

Note: For more detailed information on outputting results, see Section 23.

4.4 Modifying properties with the property editor

The **Property editor** allows viewing and, in many cases, permits modification of the properties of any object within the problem. To become familiar with this powerful feature, try the following:

- **Unlock** the project by clicking on the padlock icon .
- Click on a block (press and hold the CTRL key to select several blocks). Selected items will change colour to bright pink.
- Navigate to the **Property editor** and, using the drop-down menu,
- The properties of the selected blocks will now be shown - try changing the unit weight.
- *The bridge will reset so that a new analysis can take place that will account for any changes that have been made.*

- Click the **Solve** button and see the change in adequacy factor.

Before using LimitState:RING on real life projects, it is suggested that users spend time familiarizing themselves with the property editor and its functions.

*Note: For more detailed information on the **Property Editor**, see Chapter19.*

Part II

Theory

Chapter 5

Theoretical basis of LimitState:RING

5.1 Background

Masonry arch bridges are statically indeterminate compression structures which resist external applied loads primarily as a result of the thickness of the masonry and their inherent self weight. They tend to be resilient to small support movements, with these typically transforming a structure into a statically determinate form. Cracks which might accompany support movements are therefore not normally of great concern, making the notion of crack widths or other conventional serviceability criteria not applicable. Consequently engineers are generally primarily interested in guarding against the ultimate limit state (i.e. structural collapse) condition. This typically occurs when a sufficient number of hinges or sliding planes are present between blocks to create a collapse mechanism.

5.2 Analysis methods used by LimitState:RING

LimitState:RING idealises a masonry arch bridge structure as an assemblage of rigid blocks and uses *computational limit analysis* methods (also known as 'plastic' or 'mechanism' methods) to:

1. analyse the ultimate limit state, determining the amount of live load that can be applied before structural collapse;
2. permit investigation of the mode of response when supports undergo small movements.

Limit analysis techniques were originally developed for steel components and structures, but it has since been shown that these can be applied to masonry gravity structures, such as piers and arches (Heyman 1982). To help understand why limit analysis theory is applicable, compare and contrast the response of a steel column with uniform plastic cross-section and a weakly mortared masonry pier, both subject to a horizontal load F, as shown in Figure 5.1.

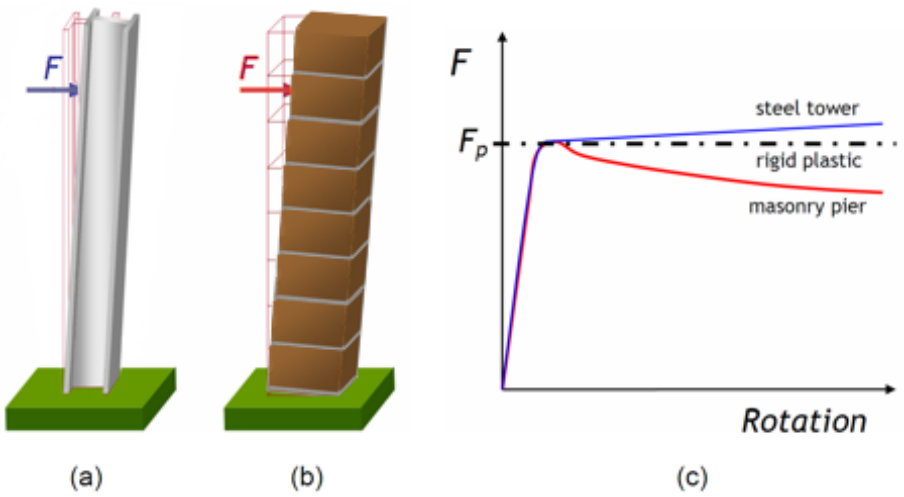


Figure 5.1: Laterally loaded (a) steel column, (b) masonry pier, and (c) idealised response curves

It can be deduced that:

- whilst the tensile and compressive strength of the steel column endow it with a finite plastic moment of resistance, M_p, the absence of tensile strength means that the masonry pier does not possess a comparable moment capacity derived from material strength;
- however, the thickness and self weight of the pier mean that there is some resistance against overturning and the masonry pier could conceptually be considered as possessing a moment capacity, albeit one that varies with height (moment capacity equal in magnitude to the normal force at a given cross-section multiplied by half the pier thickness);
- furthermore, provided pier displacements do not become large, the resistance of the masonry pier against overturning at a given cross-section will remain broadly constant;

© LimitState Ltd

- hence, the response of the pier can be considered 'ductile', an important requirement in order for limit analysis theory to be applicable.

As already indicated, in masonry structures the moment of resistance effectively varies continuously and consequentially this can make conventional bending moment diagrams difficult to interpret. It is normally more useful to plot the eccentricity of the compressive force, or *thrust*, at each cross-section (where eccentricity = moment / thrust). Figure 5.2 shows the resulting *lines of thrust* at collapse for two different configurations of masonry blocks and loading types:

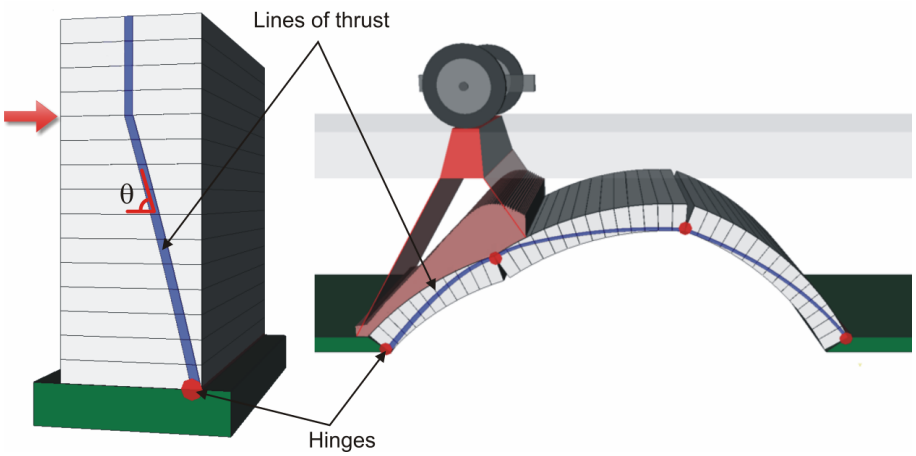


Figure 5.2: Thrust line at collapse in (a) masonry pier, and (b) masonry arch

In Figure 5.2 it can be seen that the lines of thrust lie entirely within the masonry and also that *hinges* form at the locations where the lines of thrust touch the exterior faces of the blocks. Formation of a sufficient number of hinges (and / or planes of sliding) leads to collapse.

In the case of the masonry pier shown in Figure 5.2(a), the structure is *statically determinate* and statics alone may be used to uniquely determine the position of the thrust line both prior to and at ultimate failure. In contrast, the masonry arch shown in Figure 5.2(b) is *statically indeterminate* and there are many possible positions of the thrust line prior to failure. Therefore, the actual position can only be uniquely determined at the point of ultimate failure.

In addition to basic equilibrium considerations, in the context of masonry gravity structures, the following conditions may be used to test for ultimate collapse (assuming both hinging[1] and sliding[2] failures at masonry joints are considered possible):

1. The *yield condition*, which may be deemed to be satisfied providing the line of thrust both lies entirely within the masonry and does not cross any joint at a subtended angle (θ) of less than $\tan^{-1}(\mu)$, where μ is the coefficient of friction.

2. The *mechanism condition*, which may be deemed to be satisfied providing the line of thrust either touches exterior faces of the masonry blocks and/or crosses sufficient joints

[1] Initially assuming that the masonry possesses infinite compressive strength, so the line of thrust can be transmitted through a hinge point lying on an exterior face of the arch.

[2] Initially assuming that sliding failures follow a 'sawtooth friction' model (i.e. obey an associative flow rule, where sliding is accompanied by dilatancy).

© LimitState Ltd

at an angle (θ) of $\tan^{-1}(\mu)$ to create the releases required to transform the structure into a mechanism.

Thus, if a line of thrust satisfies the *equilibrium* and *yield* conditions, then the true plastic collapse load cannot be less than the applied load – i.e. it is a lower bound.

Similarly, if a line of thrust satisfies the *equilibrium* and *mechanism* conditions, then the plastic collapse load cannot be higher than the applied load – i.e. it is an upper bound.

It is possible to perform limit analysis by hand. For example, an upper bound hand analysis could be carried out by:

1. choosing a likely mechanism of collapse;
2. using equilibrium (or a work method) to calculate the collapse load;
3. trying other likely collapse mechanisms until the critical one is found.

However, even in the case of a single span, single ring, arch the curved geometry makes such a hand based procedure extremely laborious and liable to human error. LimitState:RING effectively automates this process. However, rather than adopting a trial and error procedure to find the mechanism associated with the absolute minimum collapse load, LimitState:RING uses rigorous mathematical optimisation techniques to quickly and accurately determine the correct solution (refer to Appendix A.1) for details of the mathematical problem formulation).

Furthermore, whilst the above discussion has implied that masonry possesses infinite compressive strength, this is clearly not the case in reality. In fact the presence of finite strength masonry means that the line of thrust mentioned previously is better thought of as a *zone of thrust*, which should have sufficient thickness at any point to carry the compressive force, with the required thickness depending on the crushing strength of the masonry. Clearly, this consideration would add extra complexity to an already tedious hand calculation. However, if a finite masonry crushing strength is specified by the user then this thickness is automatically computed by LimitState:RING. It is assumed that the thrust is carried by an area of material under a uniform level of stress (i.e. assuming a rectangular stress-block, in accordance with a rigid-plastic idealisation of the masonry crushing response).

© *LimitState Ltd*

5.3 Output from a LimitState:RING analysis

5.3.1 Identification of the adequacy factor

Although for simplicity the previous section considered a case where a collapse load (e.g. F_p) was to be computed, it is generally more useful to compute the factor which would, when applied to some specified pattern of live loads, lead to collapse. This factor (or 'multiplier') is commonly termed the 'adequacy factor' and its determination for a given bridge is the principal goal of a normal LimitState:RING analysis. When appropriate partial factors are included in the model, this adequacy factor must be greater than 1.0 for a safe structure[3].

For example, if a 1kN single axle load is specified and LimitState:RING indicates a computed adequacy factor of for example 154, this means that the load which would cause collapse is 154kN. Alternatively if a 100kN single axle load was specified, the adequacy factor computed would be 1.54 (1.54$\times$100 = 154kN).

When the applied load comprises a series of axle loads, the adequacy factor is the multiplier which, when applied to all axle loads simultaneously leads to collapse. For example, if a 1400kN rail vehicle comprises four 200kN axles and four 150kN axles and LimitState:RING indicates a computed adequacy factor of 3, this means that the loading at failure comprises four 600kN axles and four 450kN axles (3$\times$200 = 600kN; 3$\times$150 = 450kN).

Full details of the mathematical formulation are provided in Appendix A.1.

5.3.2 Bridge behaviour when subjected to support movements

Although the usual ('standard analysis' mode) goal of a LimitState:RING analysis is to identify the adequacy factor, LimitState:RING can also be used to model the effects of support movements. Movement of a support will lead to formation of hinges and/or sliding failures in the same way as when excessive applied loading is applied by a highway or railway vehicle.

When in 'support movement analysis' mode, vehicle loads (if present) can be pre-factored by the user and the critical arrangement of hinges and/or sliding failure planes are identified by finding the energy associated by moving the supports. The line / zone of thrust is also identified. Figure 5.3 shows the result of a support movement analysis. Full details of the mathematical formulation are provided in Appendix A.2.

[3]The adequacy factor may need to be commensurately greater than 1.0 for a safe structure if dynamic and/or other factors have still to be applied following completion of a LimitState:RING analysis.

© *LimitState Ltd*

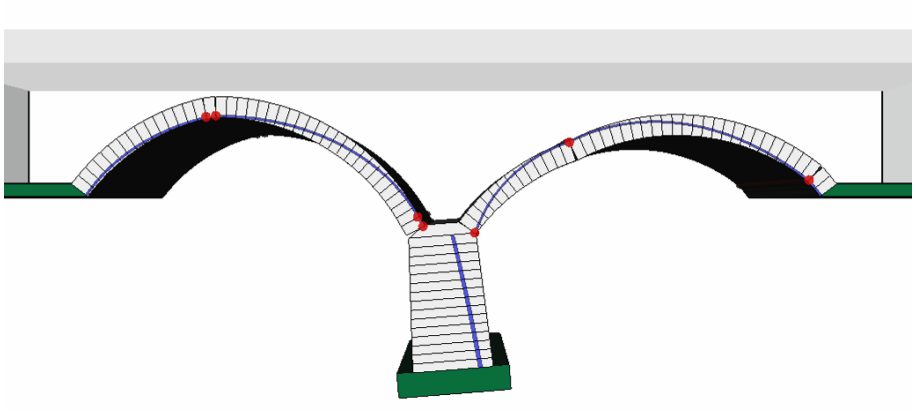


Figure 5.3: Modelling the effects of support movement at the base of a pier

© *LimitState Ltd*

5.4 Range of applicability of LimitState:RING

5.4.1 Spans

When the adequacy factor is sought, LimitState:RING is most suited to the analysis of single and multi-span masonry arch bridges with short to medium span lengths, where foreseeable live loadings are typically non-negligible in comparison to structural self-weight.

For spans longer than 20 to 30m, foreseeable live loadings are often essentially negligible and other considerations become more important, such as long term masonry creep effects (due to persistent moderately high stresses). Additionally, in the case of very long span bridges, the presence of high compressive stresses may give rise to non-negligible second order deformations, making the adequacy factors computed by LimitState:RING potentially non-conservative.

5.4.2 Block shape

LimitState:RING provides a realistic model of bridges comprising single or multi-ring arch barrels constructed using regular stone blockwork or brickwork. Since in LimitState:RING the constituent blocks are assumed to be rectangular, the software provides a less realistic model of random rubble stone masonry arches. Nevertheless, given that it is unfeasible to model the actual layout of stones in a random rubble arch, the software may be applied to such arches provided suitably conservative smeared properties for the masonry are adopted.

5.4.3 Stress-related failures

In common with other limit analysis (or 'mechanism') programs, LimitState:RING may not accurately predict the ultimate strength of a bridge if either of the following apply:

1. The bridge comprises a long (e.g. $>$ 20 - 30m) and/or flat arch (e.g. span/rise $>$ $\approx$6, or the arch contains very flat sections, for example in the case of an elliptical arch) and it can be expected that elastic deformations prior to collapse will significantly change the arch geometry;
2. A brittle response of some part of the structure may be expected to prevent the formation of a ductile collapse mechanism (e.g. abrupt failure of the bond between rings; brittle hinge formation).

However, even in these cases LimitState:RING can provide an invaluable upper bound estimate of the likely strength of the bridge, which can be used as a benchmark for alternative analysis methods. Such alternative methods may comprise: in the case of (i) a geometrically non-linear elastic analysis; in the case of (ii) a non-linear elastic analysis incorporating a masonry material model respecting fracture mechanics principles (e.g. using Hillerborg's cohesive crack theory, Hillerbourg 1976).

Alternatively if in the case of (ii) the brittle response stems from shear failure at masonry joints, it may alternatively be assumed that the initial bond strength at these joints is zero, and that

only compressive and frictional forces may be transmitted. In this case LimitState:RING may be used and can normally be expected to provide lower bound (conservative) results.

Unfortunately the decision as to when cases (i) and (ii) might apply is complicated by the fact that both are dependent on stress levels. If stresses are very low in comparison to the elastic modulus and/or bond strength of the masonry, then brittle fracture of the bond between rings for example is unlikely to be an important issue. Since it can be shown that stresses increase with the size of structure being considered [4], this implies that brittle fracture of the bond between rings may not be of great concern in the case of very short span bridges. Readers are referred to Section 7.8.2 for further discussion on modelling multi-ring arch bridges.

5.4.4 Fill depth

Additionally, since LimitState:RING has been calibrated in situations when fill depths are relatively small in comparison to the arch span, for bridges with relatively small fill depths at the crown (or with no fill) LimitState:RING can be expected to provide reasonable results. Conversely, when the fill depth at the crown is large (e.g. $>$ span/2) results from the program should be treated as being very approximate. In such cases it may often be found that the predicted load carrying capacity is in excess of the bearing capacity of the arch fill material (i.e. is unattainable in practice).

5.4.5 3D effects

In general spandrel walls at the edges of a bridge can stiffen the arch prior to failure and, depending on their end restraint conditions, may also enhance the ultimate limit strength. Studies of the influence of spandrel walls on the carrying capacity of full-scale single and multi-span laboratory bridges are detailed elsewhere (Melbourne & Gilbert 1995 and Melbourne et al. 1997).

However, if a bridge is wide in comparison to its span then the effects of the spandrel walls on the central section of the bridge may be quite minimal. Furthermore, a common defect observed in masonry bridges is detachment of the spandrel walls (this is evident by the presence of longitudinal cracks running close to the edges of the bridge). For these reasons spandrel walls are not modelled in LimitState:RING.

Since the software idealises the arch in two-dimensions, it is most suited for assessing masonry arch bridges which span squarely between abutments; readers are referred to Section 7.3 for further advice on modelling skew bridges.

By default, LimitState:RING utilises a user specified fixed bridge width in the analysis. However, an effective width which varies according to simple user-defined transverse distribution rules can alternatively be used; readers are referred to Section 8.1.4 and Section 8.2.2 for further guidance on choosing an effective bridge width.

[4] Consider for example the stresses at the base of two geometrically similar solid masonry piers. If the second pier is n times as large, in all dimensions, as the first then its volume and hence self weight will be n^3 larger. However the area at the base of the pier will only be n^2 times larger, so it follows that the gravity stresses at the base (and elsewhere) in the second pier will be $n^3/n^2 = n$ times larger.

© LimitState Ltd

5.4.6 Range of collapse modes identifiable

The general problem formulation and rigorous mathematical solvers employed mean that LimitState:RING can identify numerous potential failure mechanisms. Figure 5.4 shows a selection of those mechanisms that have been observed whilst using the program to assess real bridges.

The ability of LimitState:RING to identify hitherto unknown failure modes has led to some interesting findings. For example, it has previously been suggested that a multi-span bridge can safely be analysed as a series of separate single spans if the piers are 'stocky' (i.e. thick in comparison to their height). However, this is not in general the case. For example, the adequacy factor associated with the mechanism shown on Figure 5.4(f) is actually much lower than that computed if the outer spans are omitted from the analysis - indicating that the presence of stocky piers (and backing in this case) is no guarantee that the structure can be safely idealized as a series of separate single spans. In general, users should avoid using such rules of thumb and should where possible model as much of the structure as is practicable (refer to Section 7.4 for further advice on modelling multi-span bridges).

5.5 Use of reinforcement

The key assumptions made when modelling bridges containing reinforcement using LimitState:RING are as follows:

- The masonry (or concrete) arch and the reinforcement are assumed to behave in a rigid-plastic manner. This means that at a failing contact the reinforcement is assumed to yield at the specified force if it is located away from the neutral axis. This idealization means that the software is likely to be most suitable when sections are lightly reinforced using ductile steel bars.
- Shear failures in reinforced sections can be modelled by specifying a limiting shear force.
- Anchorage failure of the reinforcement is not considered.

For details on how to specify reinforcement in a bridge model, see Chapter 17.

© *LimitState Ltd*

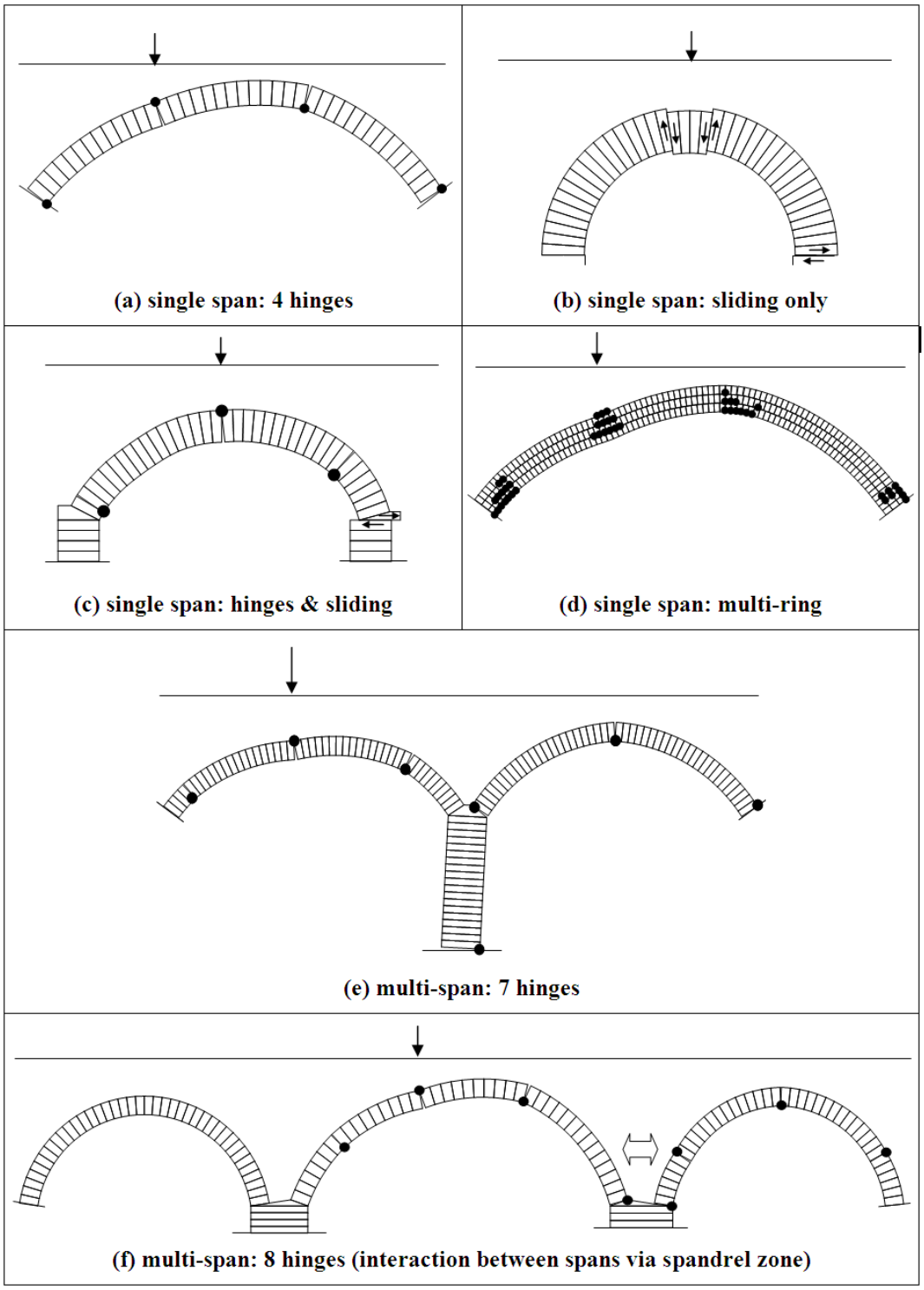


Figure 5.4: Selection of potential failure modes identifiable using LimitState:RING

5.6 Finite masonry strength

In LimitState:RING it is assumed that in the vicinity of hinges the thrust in the arch is transmitted across joints either:

1. through an infinitely thin strip of material lying on an exterior face, or, if a finite material strength is specified;
2. through a rectangular stress block located adjacent to an exterior face.

For both cases, moment vs. normal force failure (yield) envelopes for a contact can be plotted, as shown on Figure 5.5:

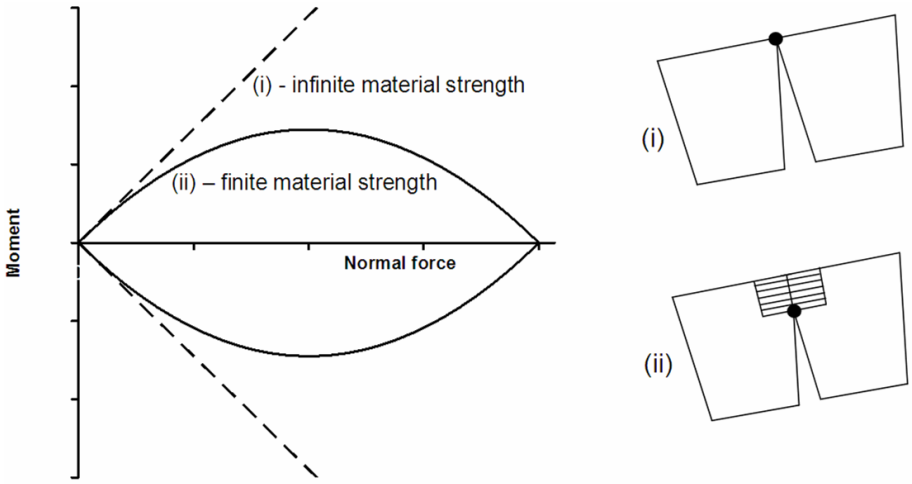


Figure 5.5: Contact surface moment vs. normal force failure envelopes for: (i) infinite; (ii) finite masonry crushing strengths

It is evident from Figure 5.5 that the envelope for case (i) is defined by linear constraints whereas in the case of (ii) these are non-linear. In LimitState:RING the envelope for case (ii) is approximated using sufficient linear constraints to ensure that any deviation from the true non-linear yield envelope is negligible. To achieve this, linear constraints are adaptively added, using an iterative procedure which is described in Appendix A.3. (The procedure ensures that converged solutions are obtained more rapidly and reliably than was the case with RING version 1.x.)

5.7 Sliding failures

Unlike many other masonry arch bridge analysis programs, LimitState:RING does not rule out the possibility that sliding failures might occur. A 'saw tooth' model for friction is used (also referred to as 'associative friction'). This means that separation is assumed to accompany sliding. The main advantage of using a 'saw tooth' model is that the linear character of the problem is preserved.

Whilst it can be shown that use of a 'saw tooth' model for friction can lead to non-conservative adequacy factors being obtained if sliding is involved in the critical failure mode (Drucker 1954), when previously applied to multi-ring brickwork arch bridges reasonably good agreement between experimental and numerical results were obtained (in fact it was found that the numerical multi-ring model always under-estimated the experimentally observed carrying capacity).

© *LimitState Ltd*

5.8 Backfill

5.8.1 General

The vertical dead weight of backfill material effectively pre-stresses the masonry in an arch, thereby increasing its load carrying capacity (provided the constituent masonry has sufficient compressive strength). The backfill also has two other beneficial effects as shown in Figure 5.6:

(i) it disperses live loads;

(ii) it can restrain movement of the arch when the latter sways into the fill. This is often termed 'passive' restraint.

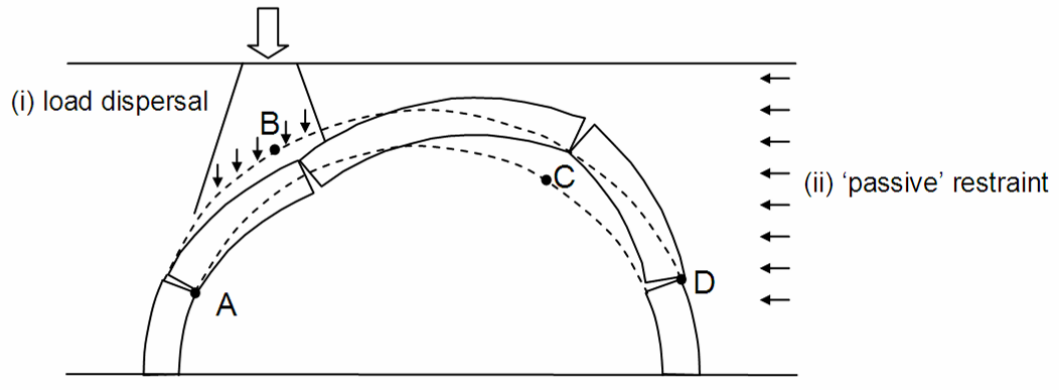


Figure 5.6: Masonry bridge soil-structure interaction

Each of the above effects has the potential to significantly enhance the carrying capacity of a masonry arch bridge. However, whereas the constituent masonry blocks of a bridge are modelled explicitly in LimitState:RING, the backfill is presently modelled indirectly according to the simplified model described in the following sections.

5.8.2 Dispersion of live loads

The vertical live load pressures on the back of an arch are assumed by LimitState:RING to be either:

1. uniformly distributed, the intensity being governed by the depth of fill under the centre of a given axle and the specified limiting dispersal angle; or

2. dispersed according to a Boussinesq type distribution, with a limiting distribution angle specified by the user.

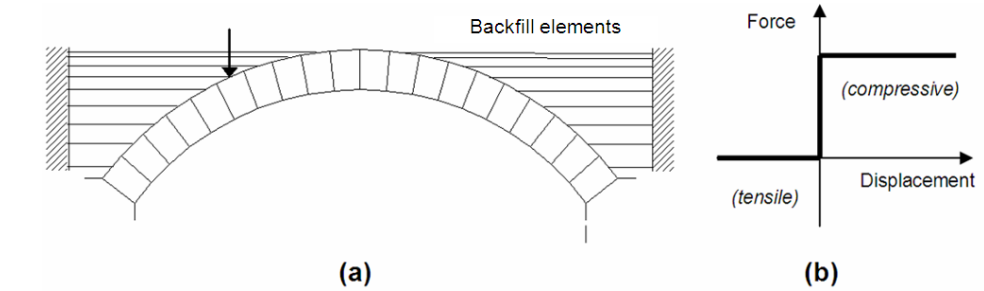
© *LimitState Ltd*

The Boussinesq distribution model generates a suitable 'bell shaped' distribution of load, which laboratory tests have indicated better approximates reality than uniform pressure distributions, and which also models the effects of overlapping dispersed loads more appropriately. This is therefore the default distribution model in LimitState:RING (except for the surface layer, if present, where a uniform distribution is adopted). A default cutoff angle of 30° is used to prevent excessive distribution. Further details of the Boussinesq model are provided in Appendix B.1.

5.8.3 Passive restraint

One-dimensional bar elements, for convenience hereafter termed 'backfill elements' (Figure 5.7(a)) are used to model the passive restraint experienced by sections of the arch moving into the fill material.

Backfill elements compress at constant force (e.g. $\sigma_h \times$ vertical projected area) where σ_h is the horizontal stress, but have no tensile capacity (Figure 5.7(b)). Use of these elements ensures that pressures are mobilised in the correct sense. For example, in Figure 5.7(c) the backfill elements only apply a force to the non-loaded side of the bridge. Note that in LimitState:RING, active pressures on the loaded side of the bridge, which are usually relatively small, are for simplicity ignored.



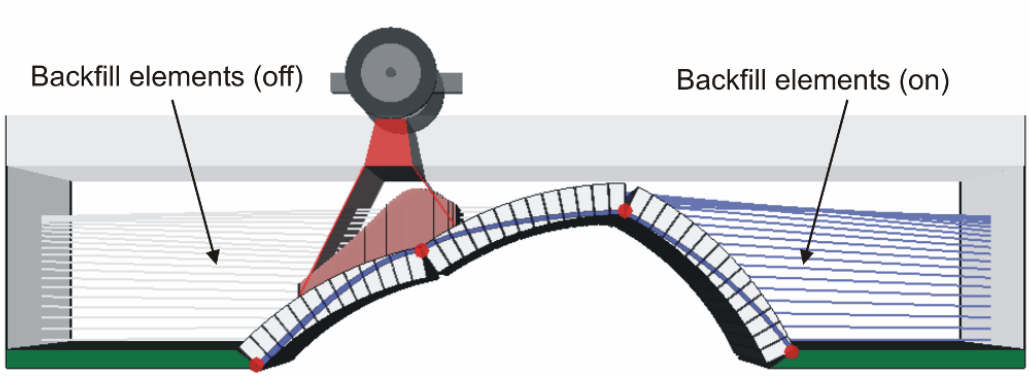


Figure 5.7: (a) Arch restrained with uniaxial backfill elements; (b) backfill element response; (c) LimitState:RING representation of backfill elements

Classical lateral earth pressure theory developed originally for vertical retaining walls is often used in masonry arch analysis to estimate the amount of horizontal passive restraint which can be realised. This approach is also used in LimitState:RING.

According to vertical retaining wall theory the horizontal passive restraining stress σ_h applied to the back of a smooth wall is:

$$\sigma_h = K_p \sigma_v + K_{pc} c \tag{5.1}$$

Where $K_p = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2(45 + \phi/2)$, σ_v is the vertical stress, $K_{pc} = 2\sqrt{K_p}$, and where ϕ is the effective angle of friction of the fill material, and c is the cohesion of the fill material. For a frictional backfill, drained strength properties ϕ' and c' would be used.

In LimitState:RING, equation 5.1 is used in modified form:

$$\sigma_h = m_p K_p \sigma_v + m_{pc} K_{pc} c \tag{5.2}$$

where m_p and m_{pc} are modification factors designed to account for additional effects not represented by simple vertical retaining wall theory:

Determination of m_p

Consider a wall which retains a frictional soil. In the case of a vertical wall which rotates at failure, full passive pressures will not be mobilised until rotations are very large (Figure 5.8(a)). In practice, a reduced earth pressure coefficient is often used to limit rotations to acceptable levels (Figure 5.8(b)). If the wall is actually curved (i.e. part of an arch) then the coefficient can be assumed to be further reduced (Figure 5.8(c)).

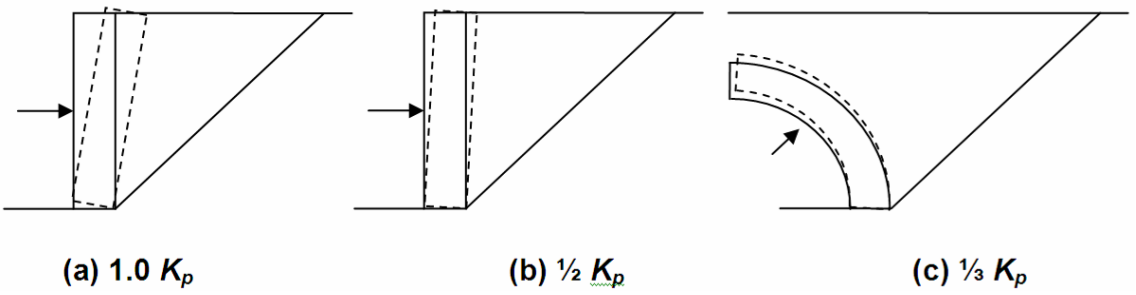


Figure 5.8: Commonly used horizontal earth pressure coefficients: (a) large wall rotation, (b) small wall rotation (≈0.5%), (c) arch segment rotation into surrounding fill

It has been found that taking $m_p = 1/3$ gives rise to a restraint force which is approximately equal to that measured in laboratory tests (at least for backfills with relatively high angles of friction). Hence this value is used as the default in LimitState:RING. i.e. when the user inputs the angle of friction for a granular backfill this is used to compute a K_p value which in turn is multiplied by the vertical stress, σ_v, and the m_p modification factor (default = 0.33) to compute the horizontal restraining stress σ_h.

Determination of m_{pc} The cohesive strength of clayey backfill materials may enhance the load carrying capacity of short span bridges. However, limited experimental evidence is available at present and a conservative default modification factor on K_{pc} of m_{pc}= 0.05 is therefore currently used in LimitState:RING. (This has been increased from the default value of 0.01 utilised in LimitState:RING 2.0 in the light of additional experimental and numerical data.) It should be noted that this value may be inappropriate for very soft, low stiffness clay backfills. However these are unlikely to be encountered in practice.

Once equation 5.2 has been used to determine a value of the horizontal restraining stress σ_h applied to a given block, this is checked to see whether it exceeds the magnitude of horizontal

stress that can be applied without causing the strip of backfill acting on the block to slide. If so, then the reduced horizontal stress associated with the sliding failure is used (this can be overridden if a user-defined pressure is specified). Further details are given in Appendix B.2.

Finally it should be appreciated that there are several important simplifications inherent in the way LimitState:RING treats passive (and active) restraining horizontal pressures. For example: (i) backfill pressures are assumed to be mobilized by infinitesimal structural movements; (ii) the failure mode is assumed not to influence the magnitude of peak pressure mobilized. Further background information on these assumptions is provided in Appendix B.3 and Appendix B.4.

5.8.4 Backing

LimitState:RING permits the modelling of backing material above the abutments and piers (see Chapter 13).

The backing model is a special implementation of the horizontal 'backfill element', where the compressive strength is set to a high value (5MPa by default). This allows the software to account for the transfer of compressive forces between (or away from) spans where strong material is present.

If desired, the resistance offered by the backing can be tailored on an element-by-element basis, allowing the software to represent differing bridge backing conditions. Details on how to achieve this are provided in Section 14.4.

Where backing exists over a pier, two backing elements will be associated with a block (one element from each of the two associated spans). The allowable force that the backing is permitted to assume in these cases is the average of the two values. Note that, when two adjacent spans have identical heights and numbers of blocks, the two backing elements between corresponding blocks may lie on top of each other in the viewer. Click selecting will pick the topmost of the two and display the limiting force as appropriate.

© *LimitState Ltd*

Part III

Modelling

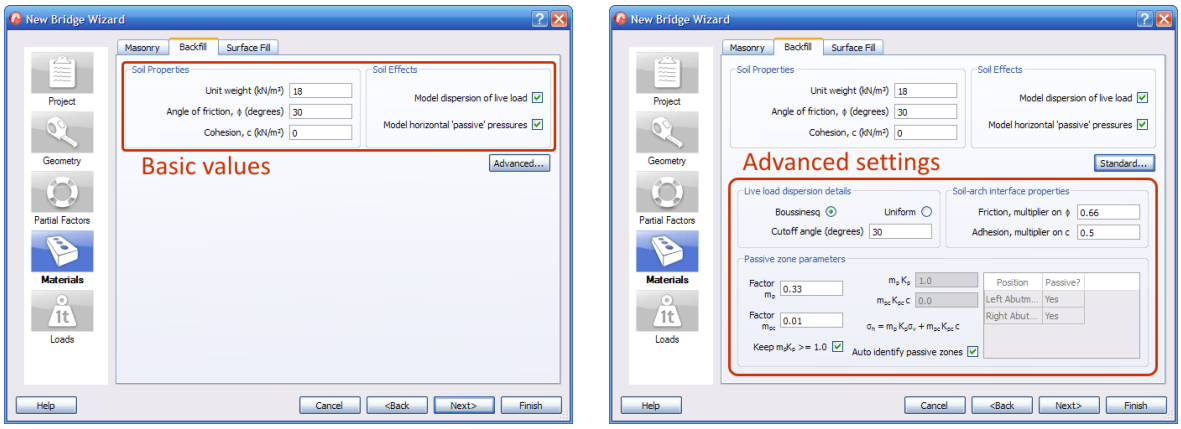
Chapter 6

Preliminary bridge assessments using LimitState:RING

Increasingly assessment codes and other guidance documents advocate a multi-level approach to bridge assessment. A preliminary (or 'Level 1') assessment might traditionally be performed using a semi-empirical assessment procedure (e.g. the MEXE method). If this assessment procedure indicates that the bridge is safe then in the past the assessment engineer has typically not been required to study the bridge further (i.e. does not need to undertake a 'Level 2' or 'Level 3' analysis).

However, there are increasing concerns over the reliability of existing semi-empirical assessment procedures, and consequently their range of recommended application is being successively reduced by the relevant regulatory bodies. The large numbers of bridges which need to be assessed therefore means that a rapid means of conservatively assessing bridge load carrying capacity is required.

Reflecting this, LimitState:RING has been designed so that it can be used with default settings to rapidly perform a preliminary and generally conservative assessment of load carrying capacity rapidly, with many dialogs in the program being simplified. For example, refer to Figure 6.1 which shows the Backfill page of the New Bridge Wizard. In a preliminary assessment default backfill parameters can be used, Figure 6.1(a). If the bridge proves to have insufficient load carrying capacity then a follow up assessment can be performed using more realistic values (e.g. informed by trial pit or penetrometer investigations). In a second level assessment it may very occasionally be necessary to also adjust other details of the ring soil model, Figure 6.1(b).



(a) (b)

Figure 6.1: Backfill dialog of New Bridge Wizard

Note that before LimitState:RING is used to perform a series of preliminary analyses it is advisable to review the default parameters, and to check that these are reasonable for the type of bridges being assessed. As an example, although the default values are normally conservative, in an area where bridges are constructed using very soft red bricks and lime mortar the default masonry crushing strength of 5MPa may be non-conservative.

Chapter 7

Detailed bridge assessments using LimitState:RING

7.1 Analysis parameters

The first step in an analysis is typically to identify sensible values for the analysis parameters. Default parameters are shown in Appendix C. However, it should be borne in mind that some of the default values proposed may be quite conservative and more accurate values should be used where possible.

Additionally, in order to save computational effort a default value of 40 is often given for the number of blocks per arch ring (rather than the actual number of units). This may lead to a very small overestimate (up to 2 or 3%) of the predicted carrying capacity of a given bridge if the real structure contains more than 40 blocks.

7.1.1 Partial factors of safety

In order to facilitate limit state analysis of bridges, values for the partial factors of safety shown in Table 7.1 can be specified.

Partial Factor Description	Symbol
Permanent load, masonry	$\gamma_{f,m}$
Permanent load, fill	$\gamma_{f,f}$
Permanent load, surface fill / ballast	$\gamma_{f,sf}$
Permanent load, track	$\gamma_{f,l}$
Axle load	$\gamma_{f,l}$
Dynamic load	$\gamma_{f,dyn}$
Material (masonry strength)	$\gamma_{m,ms}$
Material (masonry friction)	$\gamma_{m,mf}$

Table 7.1: Partial factors of safety used in LimitState:RING

Some assessment codes use a *global condition factor*. A global condition factor should not normally be applied to LimitState:RING analysis output since the effects of defects such as ring separation, low strength masonry and the influence longitudinal cracks have on the ability of a given bridge to distribute the load transversely can all be accounted for directly.

7.2 Modelling the shape of the arch

In LimitState:RING the arch shape can be modelled using one of the following profiles:

Segmental The arch profile is formed from a single segment of a circle, constructed using the crown rise and span measurements.

User-defined (multi-segment) The arch profile is formed from multiple segments of circles which fit a series of user-defined data points.

User-defined (interpolated) The arch profile is formed from a spline interpolation of user-defined data points. (This is the most powerful and flexible profile type, suitable for use when many user-defined data points are specified.)

Three-centered (pseudo-elliptic) The arch profile is (near) elliptical in shape, being formed from segments of three circles using the crown rise and span measurements.

Pointed The arch profile is pointed and is formed from segments of two circles using the quarterspan rise, crown rise and span measurements. This can therefore represent a 'gothic' arch.

It is the shape of the arch, in relation to the pattern of loadings applied to it, which governs stability. Hence it is of paramount importance that due care is taken when recording and entering the shape into LimitState:RING. All too often this is ignored, with the default 'Segmental' arch shape often being used without careful forethought[1]. When transverse cracks are present it follows that the arch profile must differ from that originally constructed, and this makes it even more important to perform an accurate survey of the bridge prior to analysis, to ensure that the true shape of the arch is modelled.

To obtain an indication of the influence of arch shape on carrying capacity, refer to Figure 7.1 which shows the effect on the computed adequacy factor of simultaneously modifying the quarter and three-quarter point rises of Bolton Bridge 3-1 (see Appendix G.1). In this case the maximum computed adequacy factor approximately corresponds to the segmental arch shape, and it is evident that quite significant reductions in adequacy factor are observed as the shape deviates from this (span and crown rise fixed but critical load position allowed to vary).

[1] Indeed it can be considered just as important to accurately record the arch shape and thickness as it would, for example, to accurately measure the overall depth, flange thickness etc. of a steel I-beam prior to assessing it's ability to span a given distance.

© *LimitState Ltd*

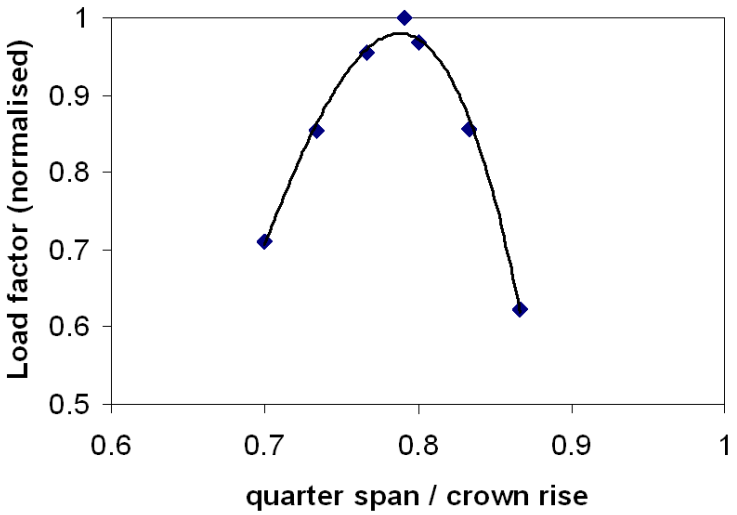


Figure 7.1: Influence of varying $1/4$ and $3/4$ point rises (as ratio of midspan rise)

It is also important to note that if the arch shape is asymmetrical (i.e. if the quarter and three-quarter point rises differ) then carrying capacity can be significantly reduced.

7.3 Skew bridges

Since LimitState:RING is 2D analysis software, it is most suitable for the analysis of bridges which span squarely between abutments. Skew bridges tested in the laboratory (Melbourne & Hodgson 1995) have exhibited distinct 3D failure modes which cannot be replicated using a 2D analysis tool.

However, given the comparative computational expense and lack of availability of mainstream 3D analysis tools, some codes of practice pragmatically permit the use of 2D analysis methods for skew bridges, and obviously LimitState:RING can be used in such cases.

7.4 Multi-span bridges

Multi-span bridges are modelled in LimitState:RING in exactly the same way as single-span structures (i.e. in essence as assemblages of interacting rigid blocks and backfill elements). Using LimitState:RING the most critical failure mode will automatically be identified, whether this involves a single or multi-span failure mode. Multi-span failure modes typically, although not always, involve two adjacent spans[2]. The following points are particularly relevant to the analysis of multi-span arch bridges:

[2]Note though that for a bridge with slender piers initial failure of one or two spans is likely in practice to quickly precipitate failure of neighbouring spans, because of the out-of-balance thrusts which then act at the tops of piers supporting these.

- for a viaduct comprising a large number of identical spans only two representative adjacent spans need normally be modelled initially;
- large railway viaducts frequently had large 'king piers' at frequent intervals (e.g. every 6 spans). These are typically sufficiently massive to ensure that no interaction occurs between the two spans abutting a 'king pier';
- for a bridge comprising spans of different geometries, ideally all spans between 'king piers' should be modelled;
- full-scale laboratory tests indicate that significant backfill pressures can be mobilised above the piers between adjacent spans, enhancing carrying capacity (Melbourne et al. 1997);
- alternatively, backing or internal spandrels are often present between spans, and this can play an important role in propping apart adjacent spans;
- in some cases the presence of strong fill or backing above a pier may mean that adjacent spans can interact in the failure mechanism without movement of the intermediate pier (e.g. see Figure 5.4(f));
- in cases where intermediate piers are very slender, the user should consider separately performing other local checks (e.g. that elastic instability will not limit the vertical load that can be applied; no such check is currently done by LimitState:RING).

7.5 End abutment blocks

Abutment blocks at the ends of a bridge can be included in the model. However, soil pressures are not applied behind end abutment blocks. Hence if end abutments are used to model soil-backed abutments there is the potential for the resulting failure mode, and adequacy factor, to be unrealistic (e.g. the skewback may be observed to slide when only a small load is applied to the bridge whereas in reality such movement would be restrained by soil).

In LimitState:RING distributed load which falls beyond an end abutment is assumed to be 'lost' (in the same way as distributed load which falls on or beyond a fixed support is 'lost'). Note that this differs from RING 1.x behaviour.

7.6 The influence of infill material

Except in the case of relatively shallow arches, the passive restraint offered by infill material (soil and / or backing) behind an arch can lead to very significant increases in carrying capacity. The problem for the assessment engineer lies in determining, for the purposes of analysis, what level of restraint is likely to be available.

In the case of a bridge with apparently insufficient load carrying capacity, it may be necessary to perform appropriate intrusive investigations (e.g. dig trial pits, perform penetrometer testing etc). These investigations can furnish soil strength parameters for use in LimitState:RING. Such investigations can also be useful in identifying unexpected beneficial construction details (e.g. the presence of generous concrete backing).

© *LimitState Ltd*

If internal hollow spandrel walls are encountered then these may be approximately modelled in LimitState:RING by:

1. specifying backing above the piers and abutments, and
2. using an averaged unit weight for the fill which takes account of both the solid and voided regions of the spandrel.

Finally, if there is evidence to suggest that the distribution of the live load is more limited than usual (e.g. due to pulverisation of the fill material) then the distribution cutoff angle should be reduced below the default value of 30^o.

7.7 Modelling the mechanical properties of masonry

In many cases the mechanical properties of the masonry are of secondary importance when ascertaining the load carrying capacity of a masonry arch bridge. Indeed, in some cases reasonable accuracy can be obtained even when it is assumed that the masonry possesses infinite compressive strength. This is most likely to be the case when the span is relatively short and when the constituent masonry is relatively strong. However, in other cases it is prudent to specify a finite masonry crushing strength in LimitState:RING.

Masonry is a composite material, comprising masonry units (stone, brick, concrete etc.) and mortar joints. The mechanical properties of the units and the joints together give rise to composite material properties which are generally quite different to those of either the masonry unit or masonry joint parent materials. An important point to realise is that the crushing strength specified in ring should be that of the composite masonry material. This will typically be lower than that of the masonry units but higher than that of the mortar in the joints.

Note also that in LimitState:RING different strengths can be allocated to the spans and piers (or even to localized areas of a span or pier).

Masonry typically fails in compression due to tensile splitting of the masonry units. Since tensile splitting is a quasi-brittle phenomenon, the compressive stress-strain characteristics of masonry do not exhibit the long flat plateau assumed in rigid-plastic theory, and also in LimitState:RING. As an example of this, the experimental compressive stress-strain loading responses of the masonry used in the Bolton test bridges discussed in Appendix G.1 are shown in Figure 7.2. To account for the potential lack of ductility, it is possible to factor down experimentally recorded values for the ultimate strength of the masonry. However, this refinement may not be deemed necessary in practice because:

1. the bridge load carrying capacity is often relatively insensitive to the precise value of the specified crushing strength;
2. the eccentric loading regime found in masonry arch bridges (in the vicinity of hinges) appears to permit realisation of higher stresses at the edge of a cross section compared with those in a specimen which has been uniformly loaded.

© *LimitState Ltd*

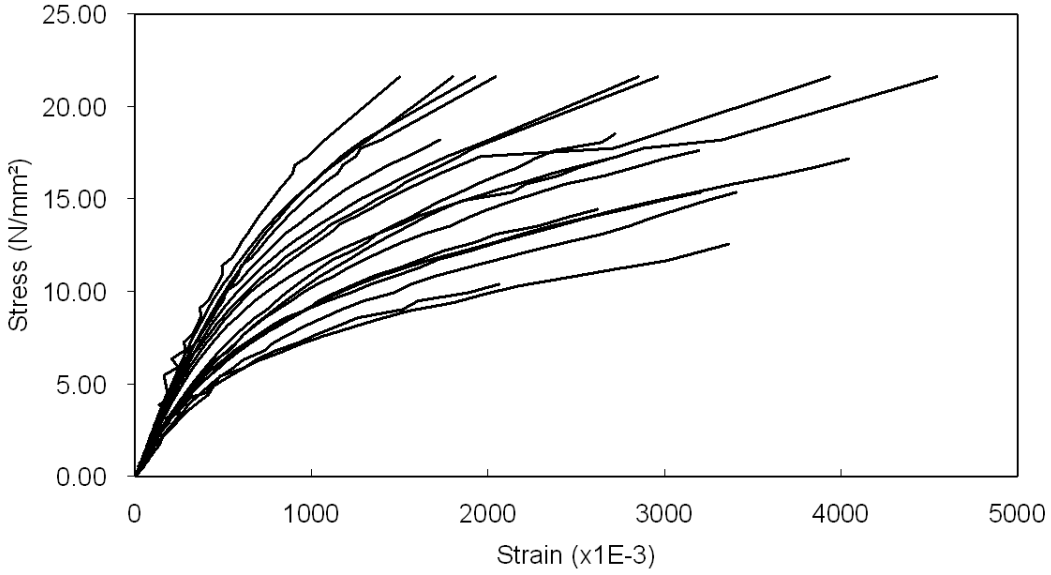


Figure 7.2: Bolton masonry compressive stress-strain loading responses (masonry: solid engineering bricks and 1:2:9 cement:sand:lime mortar)

7.8 Modelling bridge defects

The majority of masonry arch bridge stock was constructed more than a century ago and, in most cases, the passage of time will have led to a variety of material and / or structural defects. These defects can often have a bearing on the load-carrying capacity of the bridge and it is therefore important that they are modelled as accurately possible.

7.8.1 Missing mortar and/or localised spalling of masonry units

Localized areas of missing mortar or occasional spalled masonry units are inevitable in old masonry structures, and need not be considered a cause for concern. However in cases when mortar loss and/or spalled units are more widespread, and it is considered that the effective thickness of the arch barrel is being tangibly reduced, then this should be accounted for in the analysis. In LimitState:RING localized regions of arches and/or piers may be readily selected and the effective thickness reduced as required (Figure 7.3).

It should be noted that the influence of missing mortar / spalled units on carrying capacity depends not only on the depth of missing material but also on location within an arch barrel or pier. In certain locations a considerable reduction in effective thickness may be found to have little or no influence on overall carrying capacity.

© *LimitState Ltd*

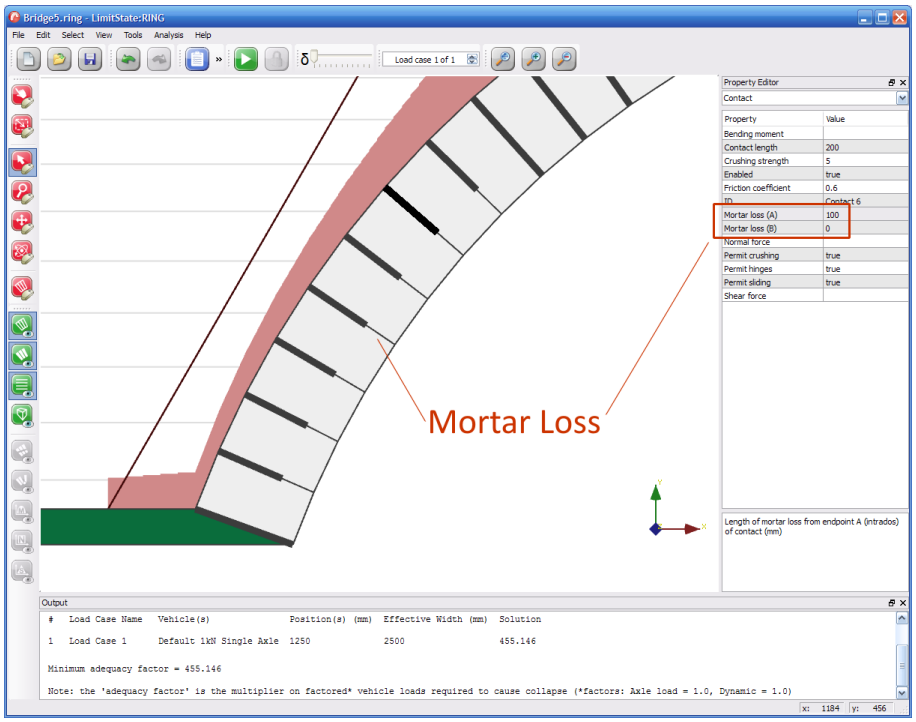
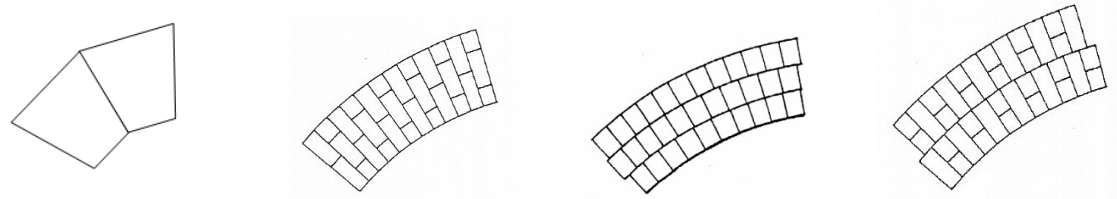


Figure 7.3: Mortar loss in a LimitState:RING model

7.8.2 Ring separation in multi-ring brickwork arches

Various bonding styles are used in the arch barrels of masonry arch bridges (Figure 7.4).



(a) stone voussoir (1 ring) (b) brick: header (1 ring) (c) brick: stretcher (3 rings) (d) brick: hybrid (2 effective rings)

Figure 7.4: Typical masonry arch bonding patterns

A powerful feature of LimitState:RING is the ability to model an arch barrel which comprises a number of separate rings (Figure 7.5).

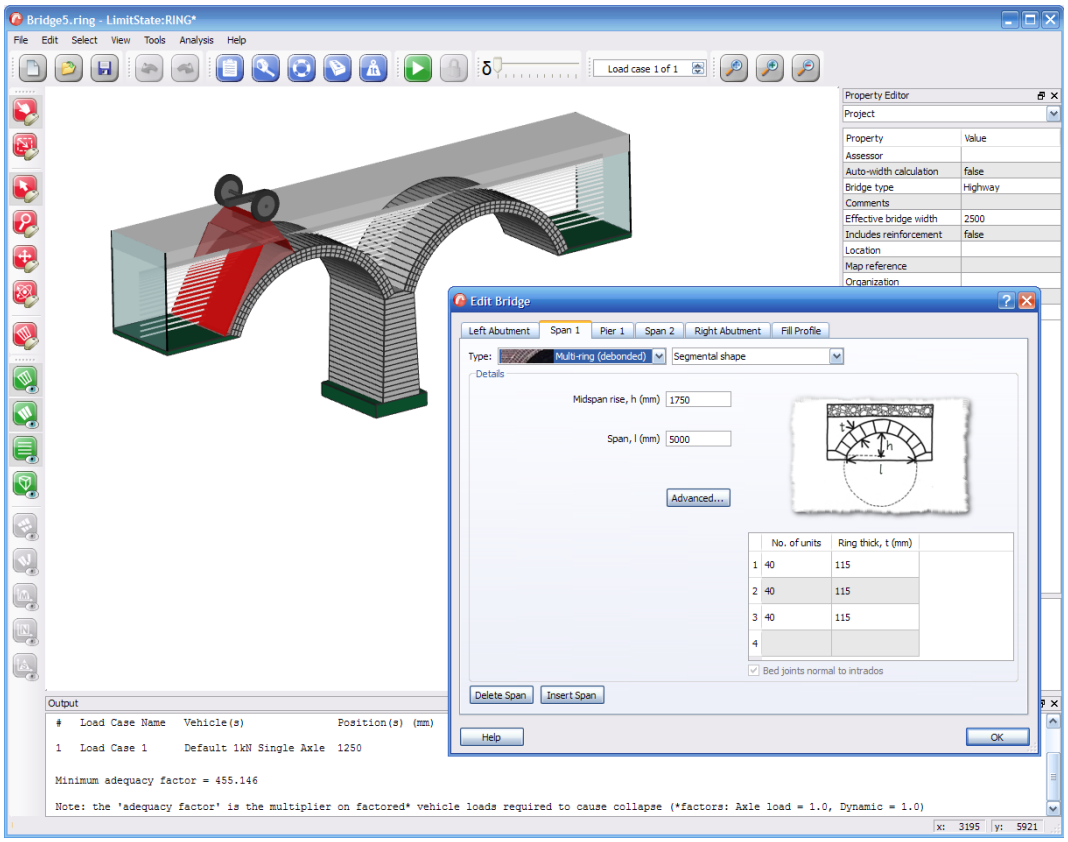
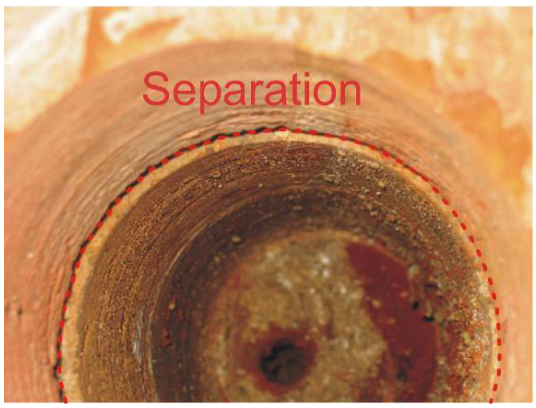


Figure 7.5: LimitState:RING model of multi-ring arch

It is usually satisfactory to assume that a brickwork arch with a bonding pattern of the form shown in Figure 7.4b effectively behaves as a single ring voussoir arch (i.e. as Figure 7.4a). However, if very weak bricks are present, headers may shear through and the arch should then be modelled as if composed from separate rings (e.g. see Figure 7.6 for a real-world example of this).

(a) view of edge of barrel



(b) view through core hole showing separation of bottom layer of brickwork

Figure 7.6: Arch barrel containing headers which have sheared through

However, even when inter-ring cracks are not evident, the circumferential mortar joints present in the multi-ring bonding patterns shown in Figure 7.4c and Figure 7.4d form potential surfaces of weakness and careful consideration should be given to modelling bridges constructed with these types of barrels.

In fact, when multi-ring brickwork arch bridges are assessed, the general question arises: should, for the purposes of analysis, the individual rings in the arch barrels of these bridges be assumed to be adequately adhered together or not? From the Bolton work referred to in Appendix G.1 the following was observed:

- Two 5m span arch bridges containing 440mm thick arch barrels, in which individual rings had been properly mortared together, suffered ring separation whilst being load tested to collapse. This separation prevented the bridges from reaching the collapse load they otherwise would have achieved (the sudden onset of ring separation was estimated to have reduced carrying capacity by up to approximately 55%).
- A similarly constructed 3m span arch bridge containing a mortar bonded 215mm thick arch barrel did not suffer ring separation when loaded to collapse.

In addition to the in-situ mechanical properties of a given joint (the above bridges were constructed using a 1:2:9 [sand:cement:lime] mortar), scaling effects will also be important in governing the likelihood of ring separation. Essentially, by almost doubling the span, rise etc, the internal stresses will also be almost doubled and for this reason very short span bridges are likely to be less susceptible to ring separation than bridges with longer spans. This can be taken account of in the analysis (e.g. a 2m span multi-ring brickwork arch in good condition may justifiably be analysed with fully bonded rings but this is unlikely to be an appropriate idealisation for a geometrically similar 20m span bridge since the higher stresses would mean it would be likely to suffer ring separation, were the bridge to be loaded to collapse).

7.8.3 Cracking in the arch barrel

Macro cracks

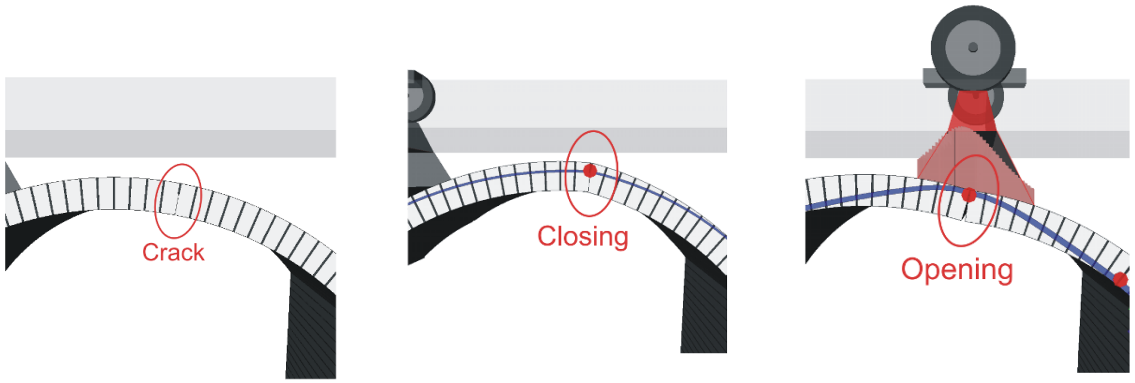
There are several distinct types of macro-cracks observed in masonry arch bridges: e.g. longitudinal, transverse and diagonal. The potential influence of longitudinal cracks on the effective bridge width and hence on ultimate carrying capacity will be briefly discussed in Section 8.1.4 and Section 8.2.2. Transverse cracks may be caused by small movements of the supports (e.g. perhaps due to slight outward spreading following removal of the centering). Identification in an arch of transverse cracks indicative of formation of three hinges is not necessarily of concern, provided the abutments are sound. This is simply the statically determinate form of an arch. However, if the location of the crown region crack/hinge is observed to change under the action of normal traffic loading then this can be problematic, with subsequent fatigue failure of the structure a possibility.

Less frequently, transverse cracks may be identified which are indicative of the partial formation of hinges due to excessive live loading at some point in the past. In general when sufficient releases (hinges and/or sliding planes) to form a mechanism are identified this should be considered to be potentially very serious. Whilst it is theoretically possible that there might exist

© *LimitState Ltd*

additional reserves of strength, for example because of the potential for increasing passive soil restraint to be mobilised as structural movements increase, in this case the large structural deformations required are such that the structure will anyway fail to meet any meaningful serviceability criteria.

It may be feasible to point up transverse cracks but if this is not done a conservative LimitState:RING analysis may be performed simply by locally reducing the arch thickness at the position of the crack, Figure 7.7 (it should be noted that LimitState:RING is not capable of modelling the presence of cracks which close up completely after finite movement).



(a) crack (b) crack closing under loading (c) crack opening under loading

Figure 7.7: Modelling an existing crack in an arch using the mortar loss feature

In square spanning bridges diagonal cracks may often be caused by abutment settlements. Whatever the cause, when diagonal cracks are present the arch profile should be surveyed at several positions across the width of the bridge, with an analysis being performed for each profile.

Micro cracks

Isolated fine cracks may be present in masonry joints or within masonry units. When numerous cracks are concentrated in parts of the structure, this can be indicative of a major problem, especially if there are signs of recent cracks. This is because masonry tends to fail in compression due to tensile splitting of the masonry units and hence widespread micro-cracking may indicate that the compressive capacity of the masonry has been almost exhausted (note that rough calculations of the stresses within a masonry pier can be misleading because the pier may be hollow or rubble filled). The cracks may continue to propagate due to long-term creep effects or due to the effects of repeated (fatigue) live loading. Masonry compression failures, although rare, will generally have catastrophic consequences and thus the assessment of a bridge with such symptoms should be carried out with extreme care; a simple LimitState:RING analysis alone is highly unlikely to be appropriate in such a case.

© *LimitState Ltd*

7.9 Modelling flooded masonry arch bridges

Masonry arch bridges typically derive the majority of their load carrying capacity from self-weight effects. In flood situations, buoyancy reduces these effects and hence also potentially the load carrying capacity. According to Archimedes principle, the ratio of normal to flooded bridge load carrying capacities may be postulated to lie between the buoyancy ratios (ratio of dry to submerged weight) of the backfill and masonry, typically 1.6 to 1.8. This has recently been confirmed by a recent series of tests on small-scale model arch bridges (Hulet et al. 2006).

When using LimitState:RING to assess an un-waterproofed masonry arch bridge subject to flooding, buoyant unit weights should be specified for the masonry and backfill materials. This enables the modelling a bridge which is flooded up to road / rail level - the most severe flooding scenario. When a bridge is waterproofed and flooded from below it is conceivable that the deleterious effects of flooding on load carrying capacity will be even more severe. This scenario can be modelled using LimitState:RING, although the hydrostatic pressures have to be entered manually (Hulet et al. 2006).

© *LimitState Ltd*

© *LimitState Ltd*

Chapter 8

Loading models

8.1 Loading from railway vehicles

8.1.1 Railway loading models

For convenience, a library containing common railway load models is distributed with LimitState:RING (see also Appendix D.1). Alternative loading models may also be defined by the user.

Some standard loading models contain components of distributed loading. Given the origins of such load models (typically determined from influence lines for simply supported or continuous beams) it is debatable as to how appropriate their use is for masonry arch bridges. Nevertheless as they are used by some authorities they are listed for completeness. However, there are two practical issues which must be borne in mind:

1. the length of distributed loading may be variable, and in this case it is the responsibility of the user to determine the length of distributed loading which proves to be most onerous;

2. the distributed loading part of a model can be introduced by adding an appropriate number of extra axles.

8.1.2 Distribution of rail loads through the track

In LimitState:RING it is assumed that a sleeper is always located under an axle load. Half of the load is then distributed to the adjacent sleepers (such that the loading follows a 25%, 50%, 25% pattern). The specified sleeper spacing governs the location of the adjacent axles (e.g. see Figure 8.1).

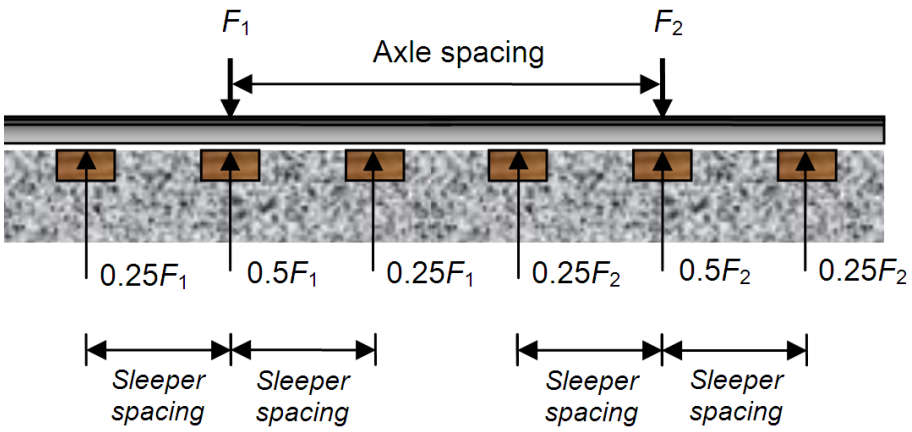


Figure 8.1: Dispersal of twin-axle loads through track

8.1.3 Longitudinal distribution of load through ballast and fill

After distributing the rail loads through the track, LimitState:RING computes the live load pressure at the underside of the sleepers. By then assuming a simple uniform distribution model this is then spread through the ballast (default spread 15°, approx. corresponding to 4:1, vertical : horizontal as per UIC 774 2R, International Union of Railways 1994). Live load is then spread through the fill according to a user specified model (uniform or modified Boussinesq distribution; refer to Section 5.8.2). Figure 8.2 shows a graphical view of how the loading from a single axle is by default assumed to be dispersed using LimitState:RING, showing different distribution angles through the ballast and backfill.

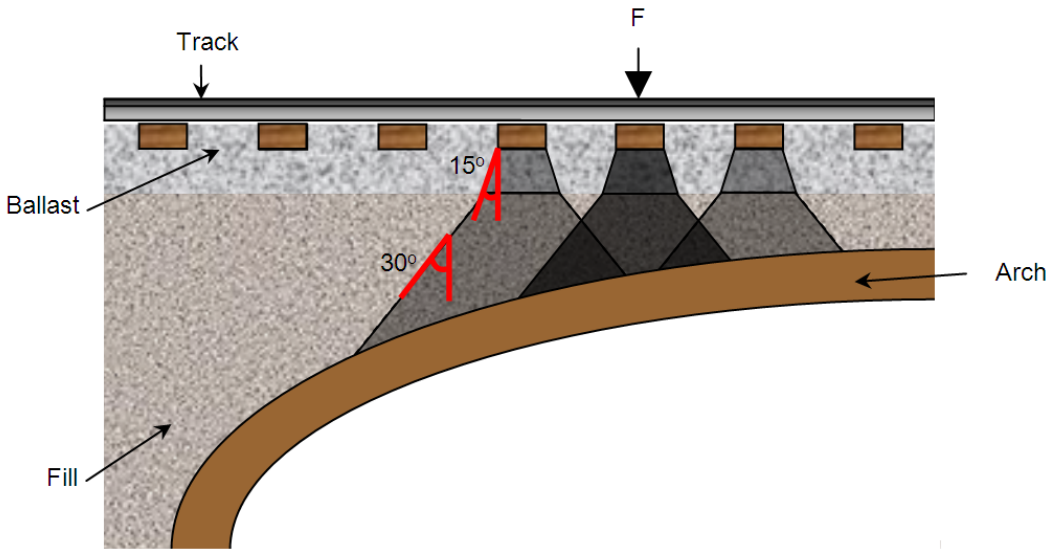


Figure 8.2: Longitudinal dispersal of a railway axle load through sleepers, ballast and fill, also showing default dispersion angles

8.1.4 Transverse distribution and effective bridge width

LimitState:RING is a 2D analysis program. Thus appropriate assumptions are required in order to determine the effective width of bridge which may be assumed to support an axle loading. Unfortunately this is an area for which there is little real evidence on which to base rational rules.

By default a fixed effective bridge width of 2500mm is used. This can be changed by the user or alternatively an automatically computed effective bridge width can be used which is computed as follows:

effective width = specified sleeper width + amount of load spread at the loaded sleeper with minimum fill depth + extra distance to account for distribution within the arch

The effective width computed using the default railway bridge parameters is shown in Figure 8.3a. However, it should be remembered that the automatically computed effective bridge width may not be reasonable and the user should check whether for example longitudinal cracks in the arch barrel, the proximity of adjacent track or the edge of the bridge will limit the effective width (illustrated in Figure 8.3b and Figure 8.3c). To address this a maximum 'cutoff' value can be specified. When this is set, the effective width will be equal to the lesser of the automatically calculated value and the specified cutoff value.

Refer to Section 12.1.2 for details on how to set the bridge width. In addition, the vertical effects of nosing and centrifugal action may mean that one rail is more heavily loaded than the other. This may mean that a concentrated wheel loading becomes the critical case and hence that a reduced effective width should be selected. Users are referred to Section 8.1.6 for further guidance.

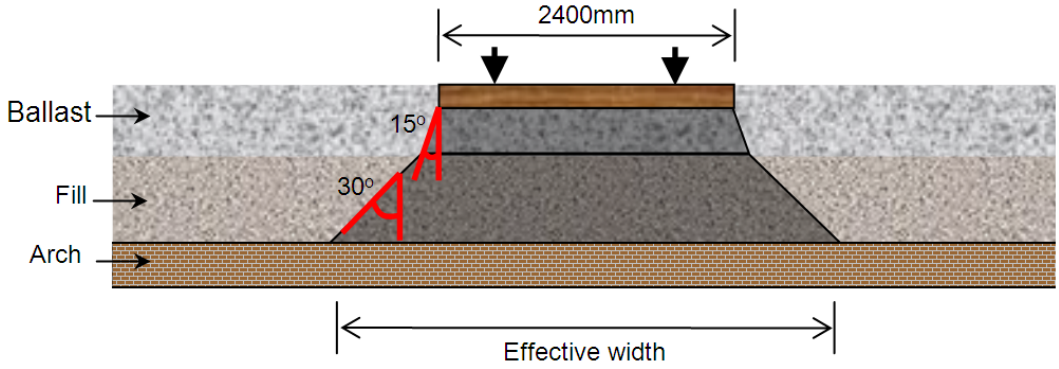
8.1.5 Dynamic effects

Underline bridges must be capable of carrying a given live train load at a given train speed. Train speed is important because certain dynamic effects are speed dependent (e.g. due to the effects of track irregularities, excitation of the bridge by the rail vehicle etc.). Thus a so-called 'dynamic factor' is applied to constituent loads in a load model.

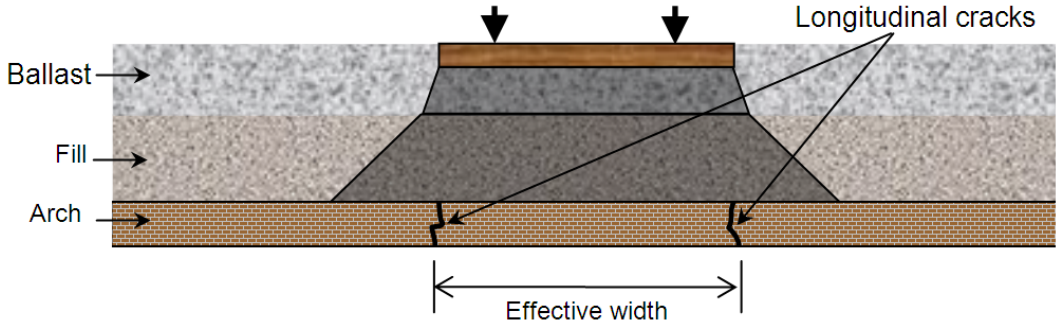
However most railway administrations stipulate that the dynamic factor is applied to all loads simultaneously. Since the pattern of loading remains unchanged, this means that dynamic effects can be considered after a LimitState:RING analysis has been completed.

In cases when different dynamic factors are applied to different axles then clearly the pattern of loading changes. This therefore needs to be taken account of in a LimitState:RING analysis. It is the responsibility of the user to apply the dynamic factor to axles in turn to determine which loading pattern is most onerous. Note that to have any effect the dynamic partial factor must also be set to an appropriate value (by default it is set at unity).

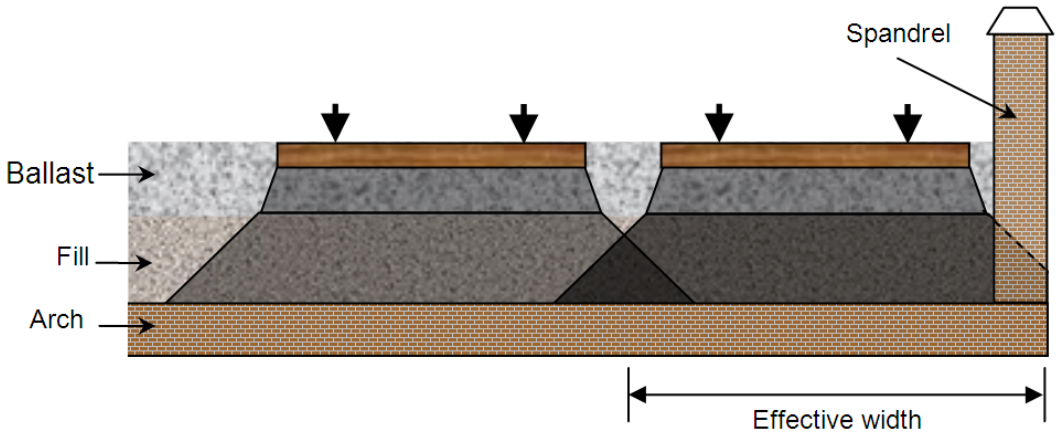
© *LimitState Ltd*



(a) Automatically computed effective width using default parameters based on sleeper width & minimum loaded sleeper ballast/fill depth



(b) Possible reduced user-specified effective width due to longitudinal cracks



(c) Possible reduced user-specified effective width due to proximity of adjacent track and edge of bridge

Figure 8.3: Transverse dispersal and effective bridge widths (railway)

8.1.6 Other effects

Nosing and centrifugal forces

On curved track the vertical effects of nosing and centrifugal actions can lead to one rail being more heavily loaded than the other. Both actions are speed dependent.

Nosing forces are caused by side contact of the wheel flange on the rail. Since the forces are generally assumed to act perpendicular to the rail, on canted track there will be a small vertical component to consider (applied to one rail only).

When one rail is more heavily loaded than the other, due to either effect, it is usually considered prudent to consider the single rail load case separately, to determine whether or not it is critical. In LimitState:RING this requires the use of a suitably reduced effective width (see Section 8.1.4). However, since the pattern of loading is unaffected then this special load case can normally be considered retrospectively (i.e. after an analysis has been performed, by modifying the adequacy factor to account for the use of a different effective width and live load intensity).

Traction/braking forces

LimitState:RING does not currently apply horizontal forces at rail level (e.g. to model traction/braking forces). However, it is possible to apply user-specified horizontal forces (as pressures) directly to blocks within arches and/or piers (see Section 19.1.2).

8.2 Loading from highway vehicles

8.2.1 Highway loading models

For convenience, a library containing common highway load models is distributed with LimitState:RING (see also Appendix D). Alternative loading models may also be defined by the user. Of the loads listed in Appendix D, the most onerous loading pattern for the majority of small to medium span masonry arch highway bridges will be that comprising a single point load.

LimitState:RING assumes that the load from an axle is spread through the surface fill (default spread 26.6°, corresponding to 2:1, vertical : horizontal as per Department of Transport 2001). Live load is then spread through the fill according to a user specified model (uniform or modified Boussinesq distribution; refer to Section 5.8.2). Figure 8.4 shows a graphical view of how the loading from a single axle is assumed to be dispersed using LimitState:RING, showing different distribution angles through the surface fill and backfill.

© LimitState Ltd

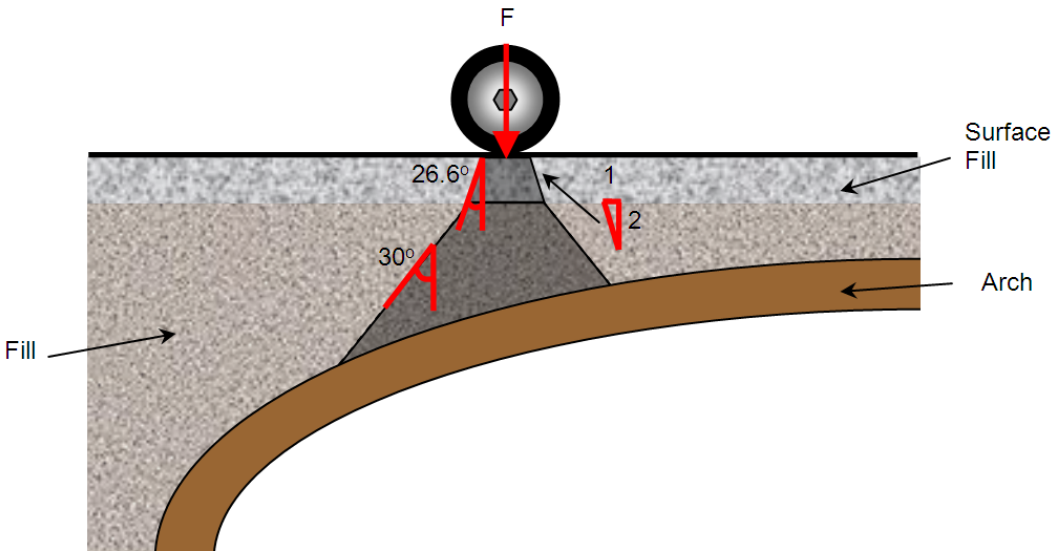


Figure 8.4: Longitudinal dispersal of a highway axle load through surface fill and underlying backfill, also showing default dispersion angles

8.2.2 Transverse distribution and effective bridge width

LimitState:RING is a 2D analysis program. Thus appropriate assumptions are required in order to determine the effective width of bridge which may be assumed to support an axle loading. Unfortunately this is an area for which there is little real evidence on which to base rational rules.

By default a fixed effective bridge width of 2500mm is used. This can be changed by the user or alternatively an automatically computed effective bridge width can be used which is computed as follows:

specified axle width + amount of load spread at axle with minimum fill depth + extra distance to account for distribution within the arch

The effective width computed using the default parameters for a highway bridge is shown in Figure 8.5a . However, it should be remembered that the automatically computed effective bridge width may not be reasonable and the user should check whether for example longitudinal cracks in the arch barrel, the proximity of adjacent lane or the edge of the bridge will limit the effective width (illustrated in Figure 8.5b and Figure 8.5c). To facilitate this, a maximum 'cutoff' value can be specified. When this is set, the effective width will be the lesser of the automatically calculated value and the specified cutoff value.

Refer to Section 12.1.2 for details on how to set the bridge width. In addition, centrifugal effects may mean that one wheel in an axle is more heavily loaded than the other. This may mean that a concentrated wheel loading becomes the critical case and hence that a reduced effective width should be selected. Users are referred to Section 8.2.4 for further guidance.

© *LimitState Ltd*

8.2.3 Dynamic effects

To account for the anticipated effects of the dynamic nature of loads applied to highway bridges some assessment codes suggest the use of a dynamic factor, to be applied to one or more of the axle loads. When a dynamic factor is applied to all loads simultaneously the pattern of loading remains unchanged, and hence dynamic effects can if necessary be considered after a LimitState:RING analysis has been completed.

Some assessment codes refer to an 'impact factor' rather than a 'dynamic factor', largely designed to take into account the effect of a vehicle travelling on an uneven road. This can be considered as a dynamic factor, though one which is generally only applied to only one of the axles of a vehicle. In this case the pattern of loading changes. This needs to be taken account of in a LimitState:RING analysis. *It is the responsibility of the user to apply the dynamic/impact factor to axles in turn to determine which loading pattern is most onerous.* Note that to have any effect the dynamic partial factor in LimitState:RING must also be set to an appropriate value (by default it is set at unity).

8.2.4 Other effects

Centrifugal forces

On a curved road the vertical effects of centrifugal actions can lead to one wheel being more heavily loaded than the other. This action is speed dependent.

When one wheel is more heavily loaded than the other it is usually considered prudent to consider the single wheel load case separately, to determine whether or not it is critical. In LimitState:RING this requires the use of a suitably reduced effective width (see Section 8.2.2). However, since the pattern of loading is unaffected then this special load case can if necessary be considered retrospectively (i.e. after an analysis has been performed, by modifying the adequacy factor to account for the use of a different effective width and live load intensity).

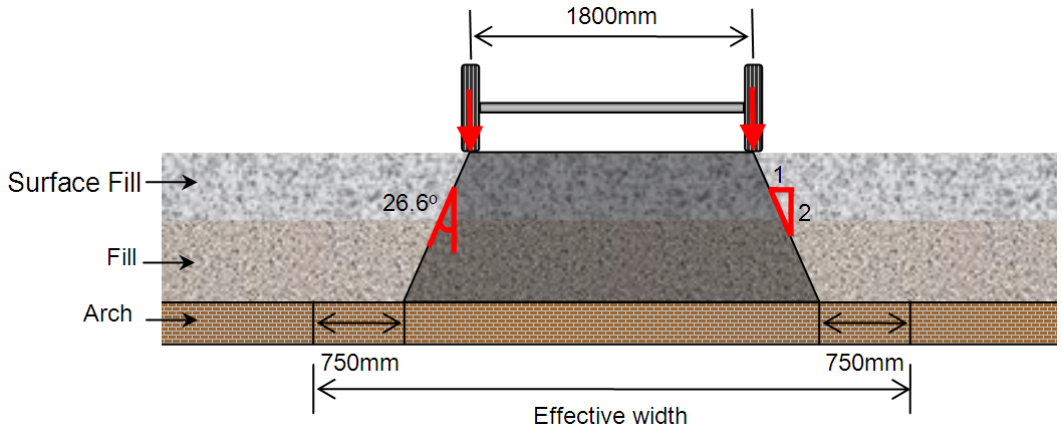
Traction / braking forces

LimitState:RING does not apply horizontal forces at road level (e.g. to model traction/braking forces). However, it is possible to apply user-specified horizontal forces (as pressures) directly to blocks within arches and/or piers (see Section 19.1.2).

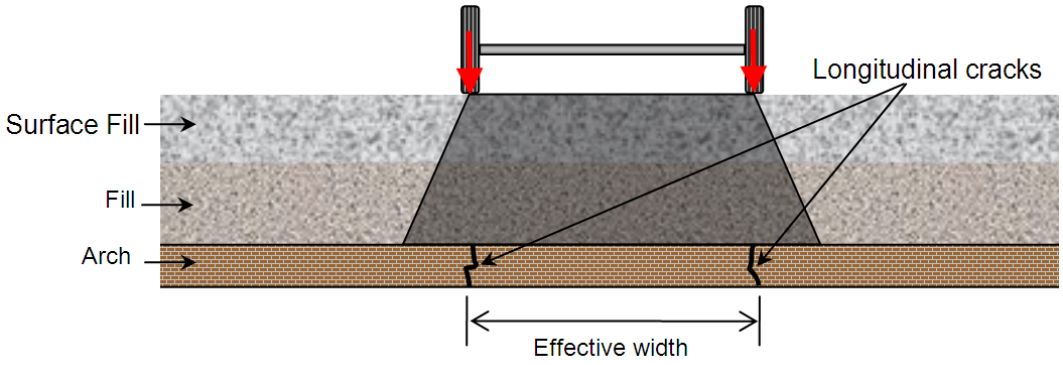
Axle lift-off

If the vertical road alignment is very sharply curved (e.g. in the case of a hump back bridge) then there is also a need to consider the possibility of axle lift off. This means that the axles which remain in contact with the bridge apply greater loading than normal. In this scenario standard vehicles can be renamed and individual axle loads edited as appropriate by the user.

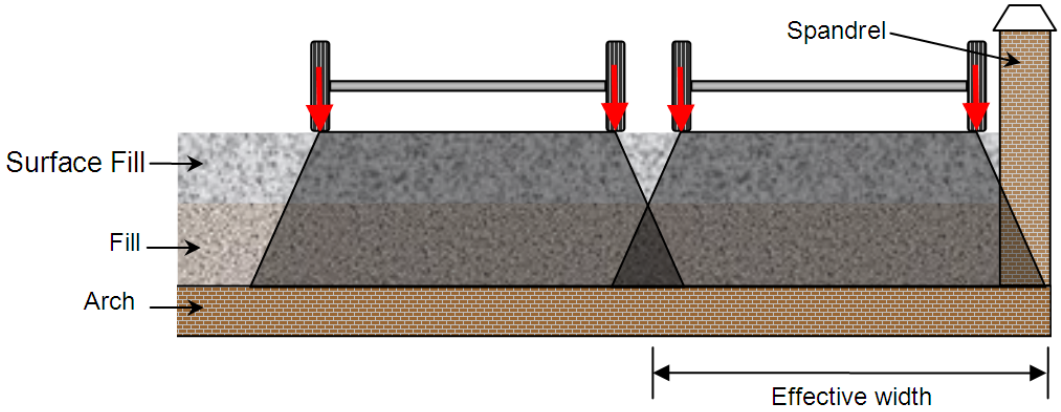
© *LimitState Ltd*



(a) Automatically computed effective width using default parameters based on axle width; minimum fill depth below axle and load distribution parameters



(b) Possible reduced user-specified effective width due to longitudinal cracks



(c) Possible reduced user-specified effective width due to proximity of adjacent lane and edge of bridge

Figure 8.5: Transverse dispersal and effective bridge widths (highway)

© LimitState Ltd

Chapter 9

Using other LimitState:RING features to investigate bridge behaviour

9.1 Identifying the causes of observed cracks using the 'Support Movement' feature

The option to model support movements in LimitState:RING opens up a range of possibilities, including the capability to investigate the likely causes of observed cracks in an existing bridge structure. By imposing support movements and comparing the actual and modelled deformed shapes, it is possible to get a sense for the various possible underlying causes of the cracking; e.g. to see whether these are consistent with vertical, horizontal or perhaps angular settlement of one or more of the piers or abutments (Figure 9.1).

The observed response of a settled bridge can also be used to verify the model idealization. A settled bridge can be considered to be of almost the same value as load test to collapse because when a bridge undergoes settlements many of the same modes of resistance are mobilized as when a bridge is subjected to excessive live loading. Therefore it is very useful to try to correlate actual and modelled behaviour (e.g. if it is necessary to include backing in the numerical model in order to replicate the observed mode of response, then this strongly indicates that backing, or very strong fill material, is present in the real structure - and potentially also in similarly constructed structures in the area. This can then be included in subsequent load factor analyses).

9.2 Exploring load paths under service loads

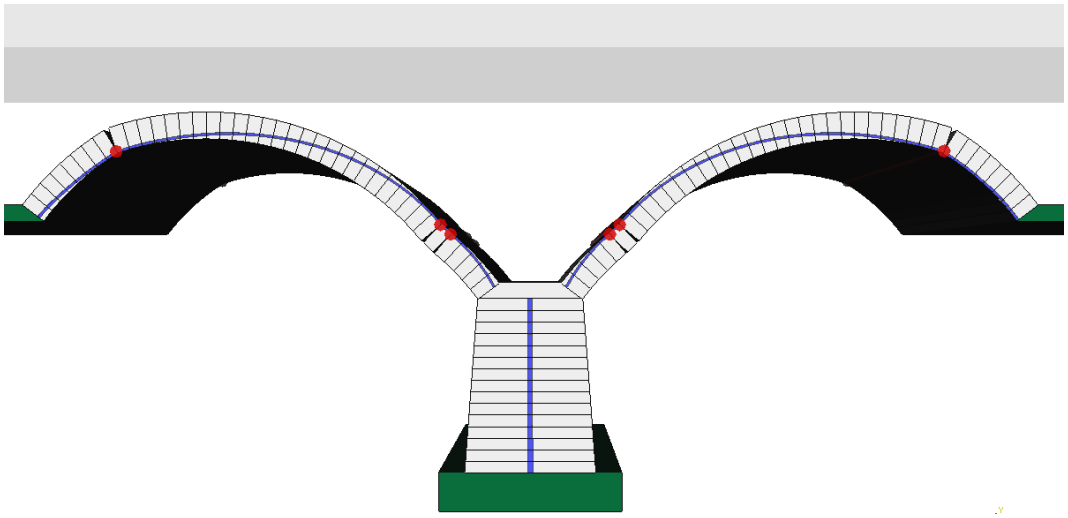
Masonry arch bridges are multiply statically indeterminate structures, and true load paths are therefore typically difficult to ascertain. It is tempting for the engineer to undertake an elastic analysis to investigate service load behavior; however, the solutions gained will only be accurate if the initial stress conditions and elastic properties are established. Otherwise misleading indications of the bridge response can be obtained.

Alternatively, using the support movement feature in LimitState:RING, various limiting scenar-

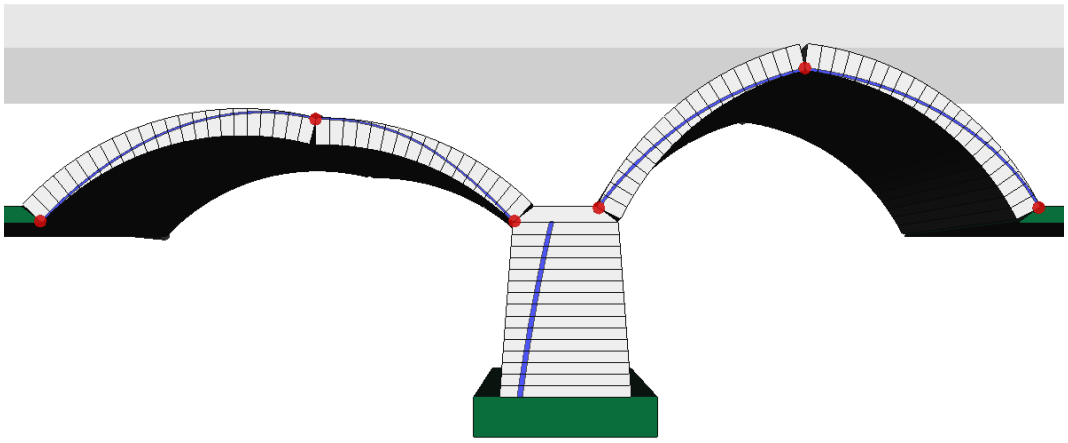
ios can be investigated. Vehicles can be run across a bridge with imposed support movements to investigate load paths and to see whether the hinge positions move (if they are predicted to move significantly in the model under traffic, and if secondary stiffening elements such as securely attached spandrel walls are not present in reality, then this might be a cause for concern as continual opening and closing of joints may lead to incremental damage to the structure).

One limiting scenario which is likely to be of interest is that which follows removal of the centering following initial construction. It is at this point that many bridges appear to bed down to a statically determinate (or near-statically indeterminate) state. This state can be approximately replicated by moving the supports appropriately in LimitState:RING. (e.g. moving the supports of a single span bridge outwards). Again, vehicles can then be introduced and load paths established. If necessary an adequate margin of safety can be ensured by applying a suitable partial factor to the axle loads, with the software indicating whether or not the structure remains stable.

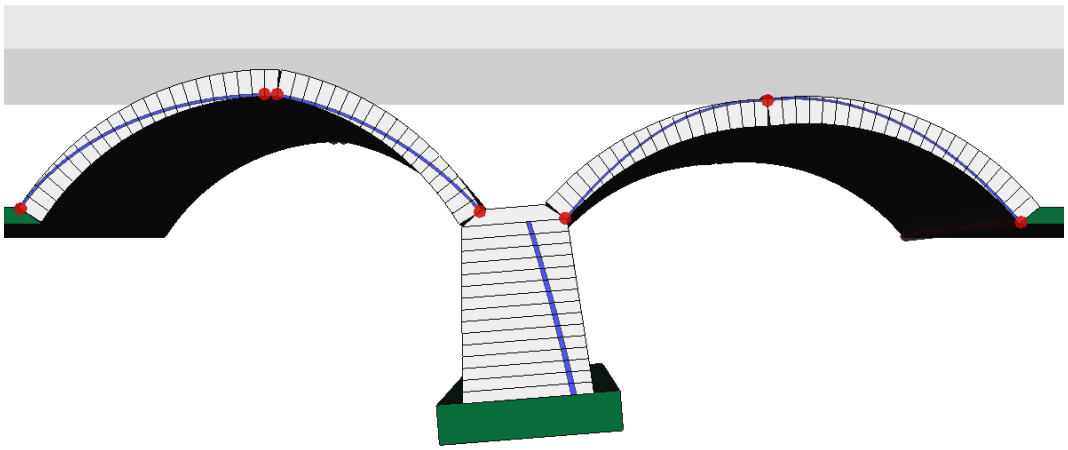
© *LimitState Ltd*



(a) Vertical support movement



(b) Horizontal support movement

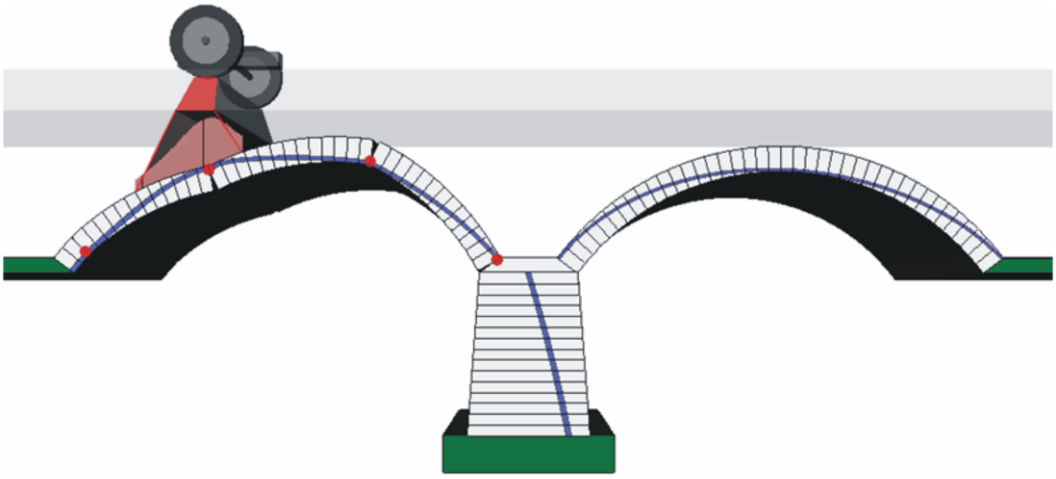


(c) Support block rotation

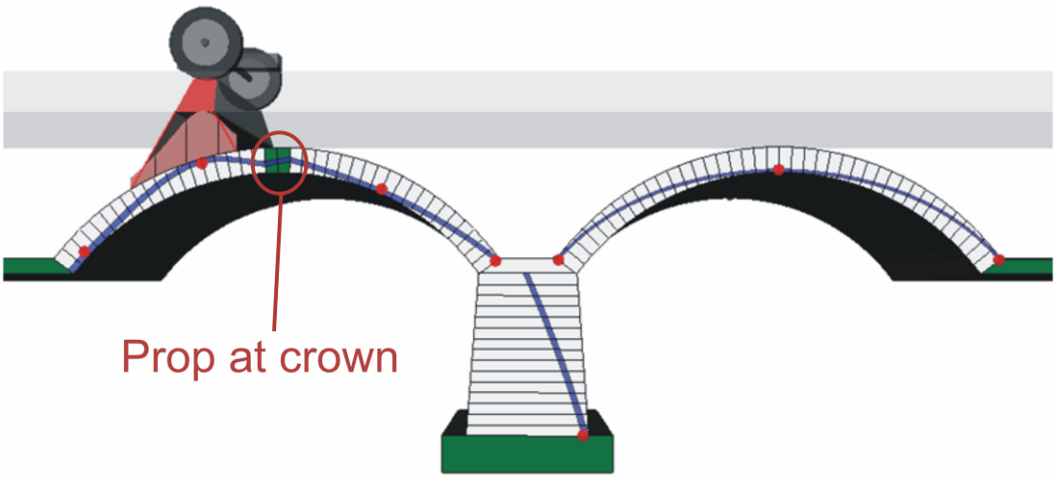
Figure 9.1: Identifying the causes of pre-existing cracks using the support movement feature

9.3 Modelling bridge spans with intermediate supports

Assessment engineers are sometimes required to analyse a masonry arch bridge which contains spans which have been propped. Whilst such propping is not in general recommended as a remedial measure, if these are present then LimitState:RING can model the likely effects of this. For example, suppose that the crown of an arch was propped. This can be modelled by specifying that one or more blocks at the crown are restrained in the vertical (y) direction. The failure mechanism is altered accordingly (Figure 9.2).



(a) unpropped failure mechanism



(b) differing failure mechanism when propped

Figure 9.2: Using LimitState:RING to mimic the effect of a midspan prop

Chapter 10

Interpreting output from LimitState:RING

10.1 Adequacy factor

The output from a standard LimitState:RING analysis is the adequacy factor, AF (see Section 5.3.1). Possible outcomes are indicated in Table 10.1.

Outcome	Scenario	Explanation
(i)	$AF < 0$	The structure is unstable action of dead loads (it would require the applied live loads to be negative [i.e. acting upwards] to stand).
(ii)	$0 \leq AF \leq 1.0$	Bridge stable under own self weight but unable to carry the specified applied live loads (able to carry $AF\times$ applied load).
(iii)	$1.0 \leq AF$	Bridge able to carry specified applied live loads (able to carry $AF\times$ applied load).
(iv)	AF could not be determined ('locked')	Structure 'geometrically locked' (e.g. for a very thick arch with infinite crushing strength).
(v)	AF could not be determined ('unstable')	Structure unstable under action of dead loads (e.g. for a very thin or distorted arch).

Table 10.1: Computed adequacy factor (AF): possible scenarios

In some circumstances an apparently adequate structure will be found to either be unstable under its own self weight, or to have a very low computed adequacy factor (i.e. Outcomes (i), (ii) or (v)). In such a case the input parameters used for the analysis should be carefully reviewed and revised if necessary. It will be found that changes to some input parameters can have a major influence on the computed adequacy factor.

Alternatively, it is possible that features not included in the model are in reality significantly altering the load carrying capacity of the structure. In this case recourse to an alternative analysis procedure may be necessary. If outcome (iv) is obtained then it is generally worthwhile

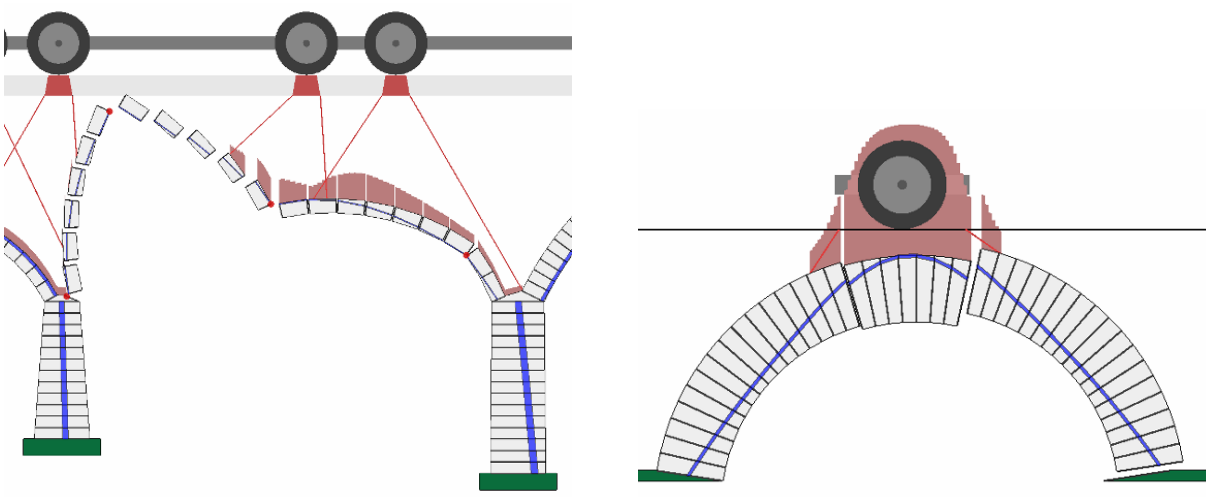
to re-run the analysis with finite masonry crushing strength so that a solution can be found.

10.2 Mode of response

LimitState:RING provides the user with a valuable visual representation of the predicted mode of response of the structure, either at failure or when support movements are imposed.

A mechanism is mobilized when sufficient releases in the structure are made. In the case of a single-span, single ring, masonry arch, the structure has 3 degrees of redundancy. This means that 3 + 1 = 4 releases are required for a 'complete collapse mechanism' (e.g. 4 hinges; 3 hinges and 1 sliding plane; etc). Multi-span and multi-ring arches are have greater degrees of redundancy and so generally require more releases. However, it is also possible for either fewer releases to be required (giving rise to an 'incomplete collapse mechanism') or for a greater number of releases to be present (giving rise to an 'overcomplete collapse mechanism'). In any event, the user should satisfy him or her self that the postulated mechanism is achievable in practice. Features of failure mechanisms:

- For a given adequacy factor the mechanism of failure is not necessarily unique (i.e. there may be two or more failure mechanisms which correspond to the same adequacy factor).
- Since the analysis is based on small displacement theory, scaling the displacements too much will lead to distortion (Figure 10.1a).
- When the mechanism includes sliding, the 'sawtooth' friction model will ensure that sliding is accompanied by dilatancy (i.e. visible separation of the blocks (Figure 10.1b). This is perfectly normal.



(a) hinging (b) sliding

Figure 10.1: Scaled deformation, leading to distortion

In some cases it will be observed that the critical failure mechanism identified cannot occur in practice (e.g. there might in reality be some obstructing element which prevents a pier from

rotating in the manner indicated in the failure mechanism). In this case the model should be modified as appropriate and the analysis re-run.

10.3 Zone of thrust / internal forces

At the point of failure the internal forces are (just) in static equilibrium with the applied dead and live loads. The most useful visual indicator of how the compressive force is transmitted through the masonry is the line of thrust. This will always stay within the masonry in order to satisfy one of the stipulated yield conditions. Additionally, forces are also transmitted through contacts between inter-ring boundaries, if present.

It is often useful to examine forces at specific locations in the bridge, for example to facilitate subsequent checks that the abutments can withstand the thrust from the arch.

Note that the distribution of internal forces is only uniquely determined in parts of the structure which are at the point of collapse. Thus although details of a distribution of internal forces for the entire structure will be displayed, it must be borne in mind that, remote from the zone of failure, this is simply one of many possible distributions.

© *LimitState Ltd*

© *LimitState Ltd*

Part IV

User Guide

Chapter 11

The Graphical Interface

11.1 Introduction

The LimitState:RING graphical interface is designed to give the user maximum flexibility over defining the problem and setting problem parameters . The default LimitState:RING screen is divided into a number of areas as shown in Figure 11.1.

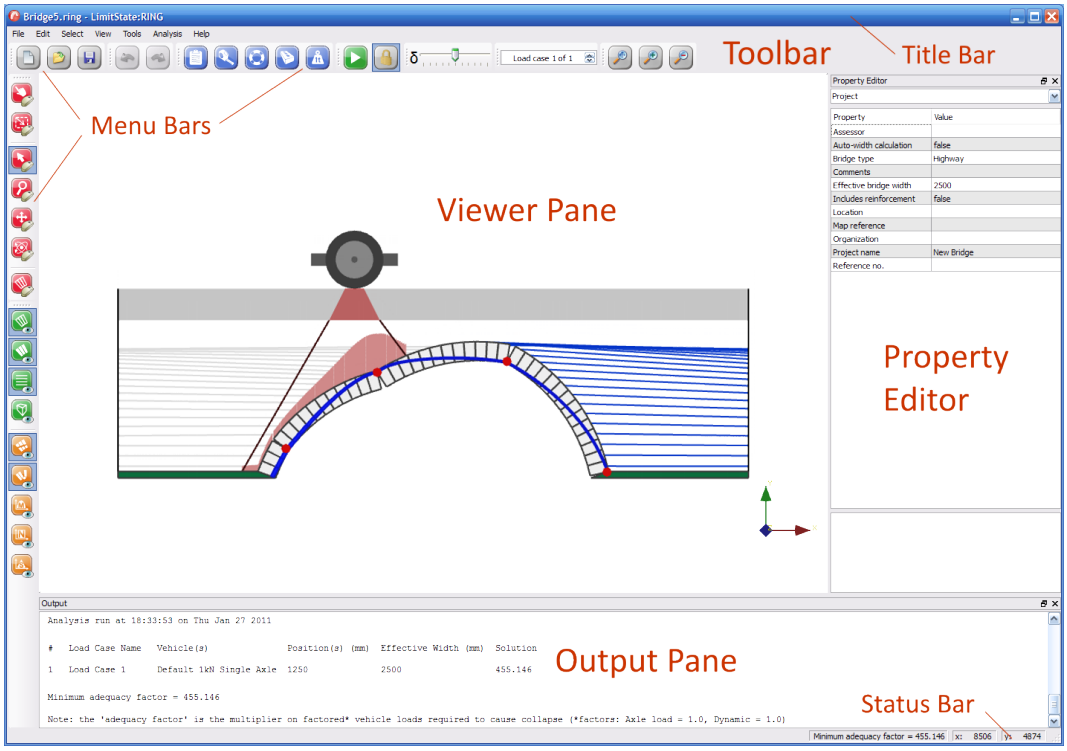


Figure 11.1: Areas utilized in LimitState:RING

The areas shown by default are as follows:

- Title bar,

- Menu bar area,
- Top toolbar area,
- Left hand toolbar area,
- **Viewer** pane,
- **Property Editor**,
- **Output** pane,
- Status bar.

A brief overview of each area is given in the following sections. A fuller description is given in later Chapters. A full list of toolbar and menu items may be found in Chapter 19.

Additional items not shown by default are the:

- **Calculator**
- **Block Explorer**
- **Contact Explorer**
- **Vehicle Explorer**
- **Load Case Explorer**

11.2 Title Bar

The buttons ***Minimize***, ***Restore Down*** and ***Close*** may be accessed by left-clicking the relevant icons at the right end of this bar or the program icon at the right. These functions may also be accessed via the context menu by right clicking anywhere on the bar.

11.3 Menu Bars

Menu bars may be selected by left-clicking on the relevant icon. There is no right-click functionality on this bar.

11.4 Toolbars

Toolbar buttons may be activated by left-clicking on the relevant button. Right clicking on any part of a toolbar brings up the explorer and toolbar selection context menu as depicted in Figure 11.2. Tooltips are available by hovering the mouse over any button for a short period. Further description of the toolbars may be found in Section 20.4.

© *LimitState Ltd*

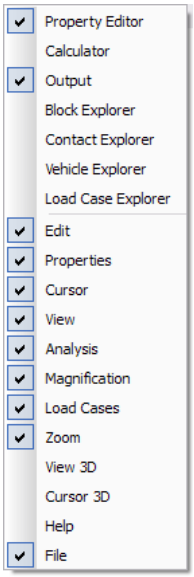


Figure 11.2: Explorer and toolbar selector

11.5 Viewer Pane

This pane displays the current problem geometry. It provides access to user editable geometry objects. Properties may be edited using the mouse or keyboard or both depending on their nature.

Specific geometry objects (e.g **Contacts** or **Solids**) may be selected by left-clicking with the mouse. The properties of the objects are then displayed in the **Property Editor** (Section 11.6).

Right-clicking the mouse in the **Viewer Pane** brings up the viewer context menu. From here, amongst other options, it is possible to modify the way in which objects are selected, manipulate the image (e.g. pan, zoom or rotate in 3D space).

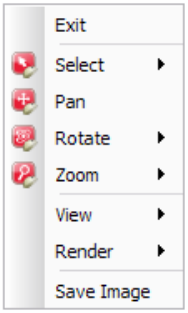


Figure 11.3: The Viewer Pane context menu

11.6 Property Editor

The **Property Editor** provides core access to problem parameters in a direct and intuitive way. In general the properties of any material or geometry object may be displayed simply by selecting it in the an **Explorer** or the **Viewer** pane. In addition, global project level parameters may be displayed at any time by left-clicking on an empty part of the **Viewer** pane with the mouse. The **Property Editor** is shown in Figure 11.4.

Single clicking on any item in the **Property** column of the **Property Editor** gives an expanded explanation of the parameter in the window at the base of the **Property Editor**.

A symbol next to an item in the **Property Editor** indicates that there are additional sub-parameters relating to that item, that may be viewed. Click on the symbol to access these. Left-clicking on a value in the **Property Editor** allows you to modify it by typing or selecting your choice (unless it is read only). For specific parameters a clickable button may also appear which gives access to a further dialog to provide additional functionality.

The calculator (see Section 11.11) may be used in any numeric data entry cell.

Right clicking on items that have been selected, brings up a context menu relevant to that item.

Further information about properties that can be edit are given in Section 19.1.

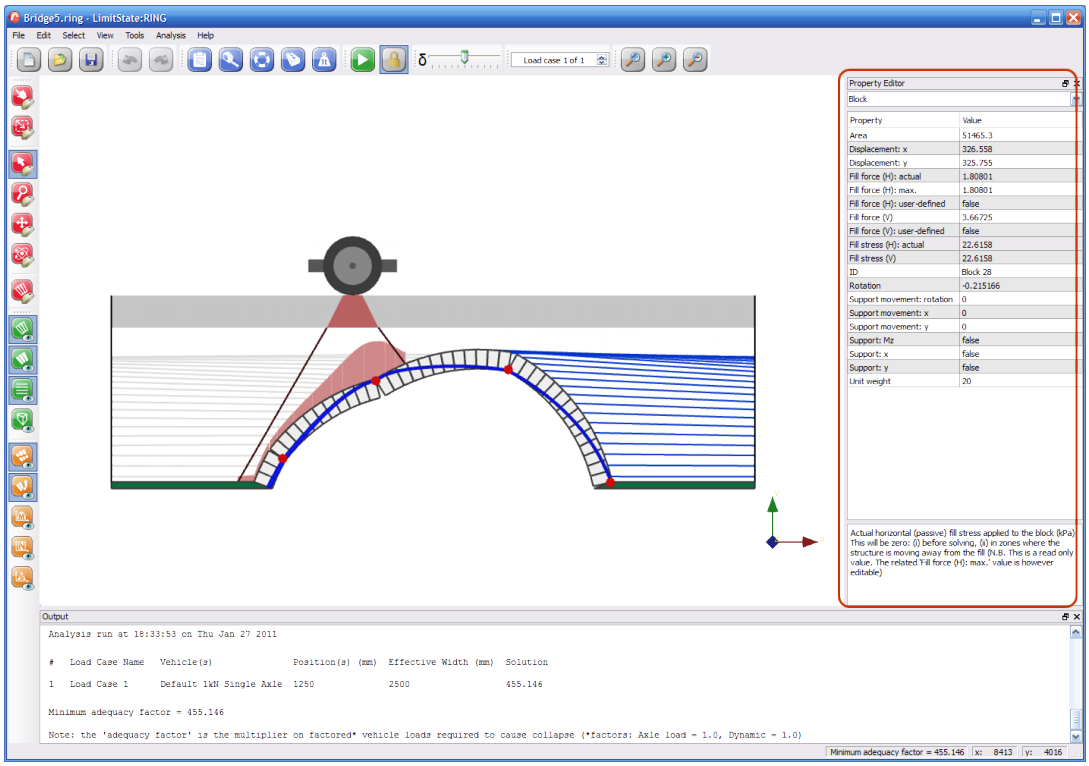


Figure 11.4: Project properties displayed in the Property Editor

11.7 Output Pane

The **Output Pane** is used to display messages and information about the analysis to the user. Any portion of text may be copied and pasted to another window/text box or other application. To select text, perform the following steps:

- left-click on the text and drag with the mouse to select a specific block of text,
- double click on a word to select it,
- triple click on a line to select it.

To copy, press CTRL-C or right click and select ***Copy*** from the context menu.

Right clicking anywhere on the window, brings up the context menu.

11.8 Status Bar

The **Status Bar** is used to provide:

- Short term progress messages to the user.
- display of the (x, y) coordinates of the current mouse position, when the mouse pointer is within the **Viewer** pane.

The **Status Bar** is used to provide:

- Short term progress messages to the user.
- display of the (x, y) coordinates of the current mouse position, when the mouse pointer is within the **Viewer** pane.

11.9 Rearranging the toolbars

It is possible for the user to rearrange the position of the toolbars within the user interface. To do this:

1. Hover the mouse cursor over the left-hand side of the toolbar you wish to move (a four-pointed arrow will be displayed).
2. Click and hold the left-hand mouse button.
3. Drag the toolbar to a new position in the interface. Available locations will be highlighted and the view will shift to accommodate the toolbar.

© *LimitState Ltd*

4. Once you have located the position that you wish to place the toolbar, release the left mouse button and the bar will be 'dropped' into place.

Many toolbars can also be shrunk to show a reduced selection of the buttons available (this is useful if you are using the software on a PC with a small screen e.g. a laptop). These are indicated by the presence of a double arrow (>>) at the end of the bar. To show the hidden icons, simply click the arrows with the left mouse button.

11.10 Detaching a menu

For convenience, the menus of the LimitState:RING interface can be detached from the main window using the 'tear off' functionality. To do this, hover the mouse over the dashed 'tear-off' line at the top of a menu and click with the left mouse button. The menu will then become detached into a separate window.

The functions of the new menu window can be used as normal and it will remain on top of the main program interface whilst still allowing interaction with it. To close the window, click the 'x' button in the top right corner.

11.11 Calculator

The calculator is used to facilitate entry of data into the **Property Editor**. If the mouse is clicked on any **Property Editor** numeric data entry cell, the calculator button appears in the right hand end of the cell. Click on this to display the calculator. The calculator (see Figure 11.5) has all the normal functionality of a standard calculator and the value that appears in the the results window of the calculator automatically appears in the data entry cell (unless it is a locked or read only cell). There is no need to copy and paste to the cell in question.

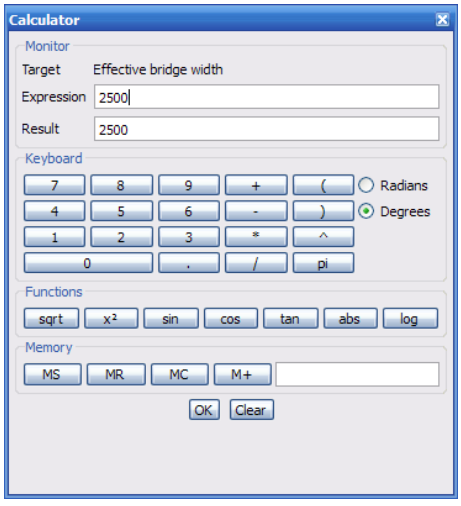


Figure 11.5: The calculator window

Chapter 12

Project details

To edit the project details, on the **Tools** menu click **Project Details**. Alternatively the command may be accessed via the keyboard shortcut by pressing **Ctrl+1** or by clicking the clipboard icon on the **Properties** toolbar. The **Project Details** dialog will be shown Figure 12.1.

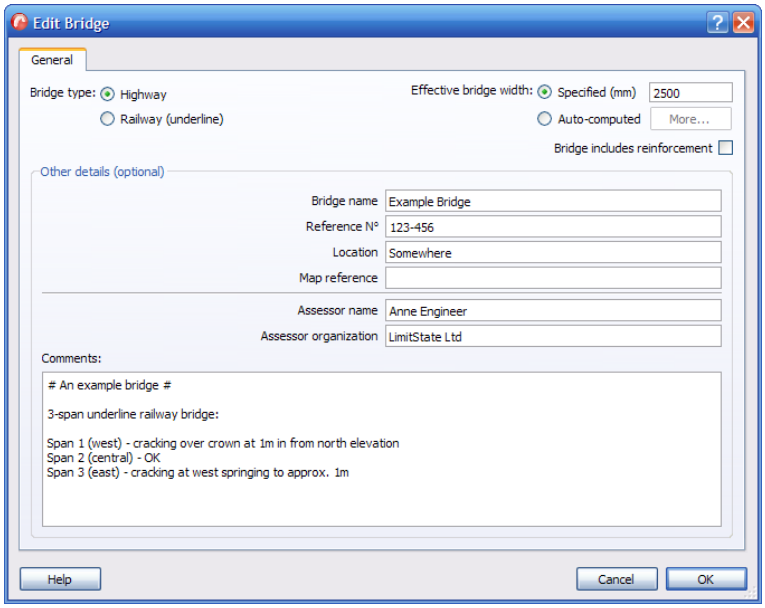


Figure 12.1: The Project Details dialog

12.1 Required details

Many of the details that can be specified in the **Project Details** section are optional. However, there are three choices that require attention:

12.1.1 Bridge type

Here the user must specify whether the bridge under consideration is subject to loading from a highway or a railway. The choice made will determine what information is required / displayed during modelling and analysis. By default, the **Highway** option is highlighted.

12.1.2 Effective bridge width

Here the user must specify whether the bridge under consideration is of a fixed width or, alternatively, if a calculated width will be used.

Loading from vehicles will tend to spread transversely as it passes through the underlying fill material. This means that a 1.8m wide load on the surface may, for example, be spread across a 3m width at arch level.

By default LimitState:RING assumes a constant specified effective bridge width of 2500mm.

However, if the user selects the option to **Auto-compute**, LimitState:RING 3.2.a can automatically calculate the effective width of a bridge according to the width of loading at the base of the fill as follows:

effective width = specified axle/sleeper width + amount of load spread at the loaded axle/sleeper with minimum fill depth + extra distance to account for distribution within the arch

By default, the effective bridge widths are calculated assuming full spread of load to both sides of the loading vehicle, using the values in Table 12.1 (see also Chapter 8):

Highway bridge 'auto-compute' defaults

The Highway bridge 'auto-compute' defaults are given in Table 12.1

Parameter	Default value
Maximum effective width	5000mm
Transverse angle of distribution through surface fill	26.6^0
Transverse angle of distribution through backfill	26.6^0
Additional transverse width of arch to include	1500mm

Table 12.1: Default values for calculation of effective highway bridge width

Railway bridge 'auto-compute' defaults

The Railway bridge 'auto-compute' defaults are given in Table 12.2

Parameter	Default value
Maximum effective width	5000mm
Transverse angle of distribution through surface fill	15°
Transverse angle of distribution through backfill	30°
Additional transverse width of arch to include	0mm

Table 12.2: Default values for calculation of effective railway bridge width

12.1.3 Model reinforcement

Here the user must specify whether reinforcement is to be modelled in the bridge. The choice made will determine whether reinforcement information is required / displayed during modelling and analysis. By default, the option to model reinforcement is not selected.

12.2 Optional details

Useful details that the user may wish to specify are:

- Bridge name
- Reference No.
- Location
- Map reference
- Assessor name
- Assessor organization
- Comments

These attributes are displayed in the ***Property Editor*** (Section 19.1) and also in the **Summary** section of the ***Report Output*** (Chapter 23).

© *LimitState Ltd*

© LimitState Ltd

Chapter 13

Bridge geometry

The bridge geometry can be specified in geometry section of the ***Wizard*** (Section 4.1.3) or by clicking on **Geometry** in the ***Tools*** menu (Section 20.3.5). Alternatively the command may be accessed via the keyboard shortcut by pressing **Ctrl+2** or by clicking the **Geometry dialog** icon on the **Properties** toolbar.

13.1 Geometry dialog

The Geometry dialog is depicted in Figure 13.1.

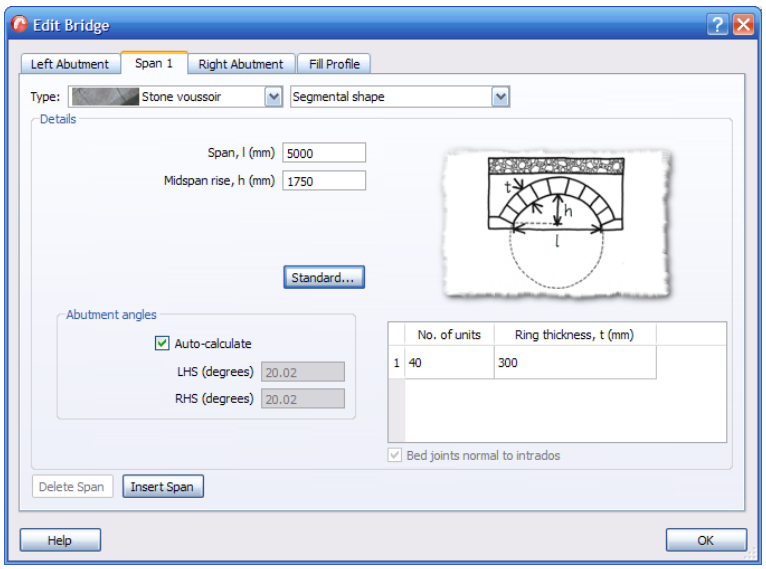


Figure 13.1: Geometry dialog box

13.2 Abutments

To edit the geometry of an abutment, simply click on the relevant tab within the **Geometry** dialog to display the dialog in Figure 13.2.

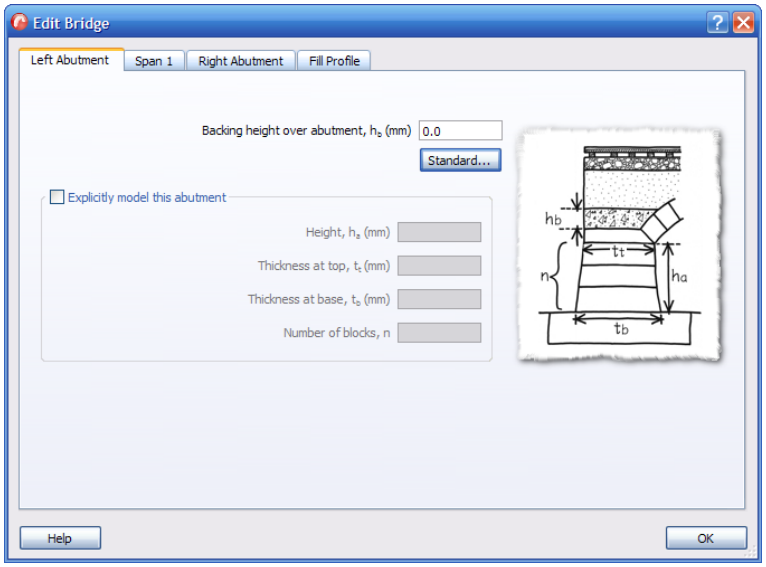


Figure 13.2: Geometry - Abutment properties

13.2.1 Default abutment model

By default, LimitState:RING presumes that all abutments are constructed similarly in that they are constructed from a single, fixed block with no backing above them.

To specify that an abutment is overlain by backing material, simply enter a suitable value in the **Backing height** box (see Section 5.8.4).

13.2.2 Modelling abutments explicitly

To override the default abutment model, click the ***Advanced*** button and check the box labelled ***Explicitly model this abutment***. The number of blocks in the abutment, and their dimensions, can then be amended.

Note: in LimitState:RING no fill pressures are assumed to act behind abutment blocks, so the explicit abutment modelling option should only be used when such fill is not present (otherwise it may reasonably assumed that the abutments are fixed).

13.3 Spans

To edit the geometry of a span, click on the relevant tab to display the dialog in Figure 13.3.

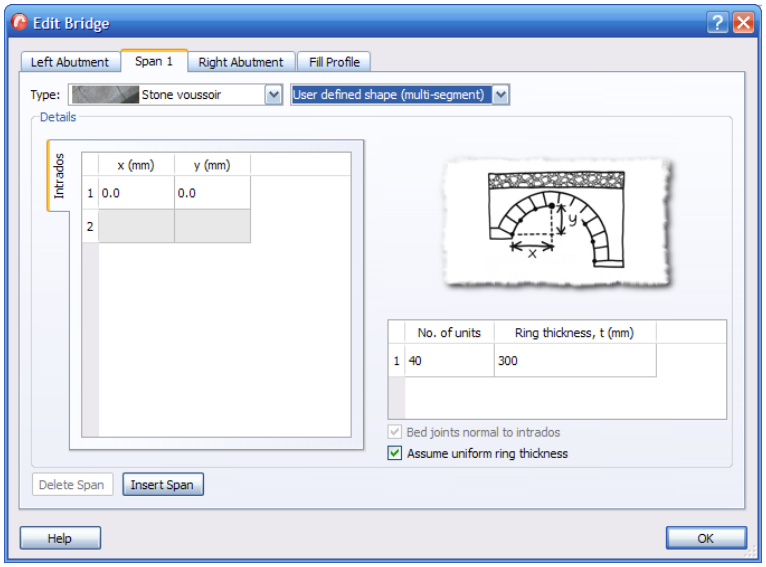


Figure 13.3: Geometry - Span properties

For all span types, there are options to specify the number of blocks (units) used for the span and the thickness of the ring.

13.3.1 Number of rings

The **Type** drop-down menu offers a choice from three types of voussoir:

Stone voussoir the span is (or acts as if it were) a single ring of stonework masonry.

Bonded brick the span consists of multiple rings of masonry that act as if they were one (e.g. if the barrel contains 'header' bonded brickwork, where certain bricks are laid 'end-on' to provide a mechanical connection between rings).

Multi-ring (debonded) the rings of a span are modelled as separate entities, separated by frictional contacts (this should be used e.g. if ring separation, also known as 'delamination' has occurred).

Note: a multi-ring analysis is often more computationally expensive than a single ring analysis.

13.3.2 Arch type

As well as the type of voussoir, LimitState:RING gives the option to select from several types of arch shape for each span:

Segmental shape A segmental arch shape assumes that the curvature of the span follows the shape of a single segment cut from a circle (defined using the span and rise measurements entered by the user). By clicking on the **Advanced** button, the user is given the option to specify the abutment angles. By default, the **auto-calculate** feature is selected. By unchecking this box, measured angles can be entered.

Multi-Segment (User-defined) A multi-segmental arch shape assumes that the curvature of the span follows the shape multiple segments formed from different circles, based on a series of user-defined points.

Interpolated (User-defined) An interpolated arch profile uses an interpolated 'best-fit' b-spline based on a series of user-defined points.

Three-centered (Pseudo-elliptic) A three-centered (pseudo-elliptic) arch profile assumes that the arch profile is formed from segments of three circles using the crown rise and span measurements.

Pointed A pointed (or 'Gothic') arch profile is formed using a segment of a circle, determined using the quarterspan rise, crown rise and span measurements, then mirrored along a vertical line at midspan.

User defined profiles

By opting for a user-defined arch profile, non-uniform shapes can be accounted for. When selected in the drop-down menu, a user-defined option will display a table in which to enter *x* and *y* coordinates of points around the intrados of the arch.

Note: the positions of all points on each arch should be measured relative to point 1 (which will have co-ordinates [0,0]). Subsequent points should be entered in order of increasing x distance.

13.3.3 Number of units

Enter the number of masonry units you wish to model in the ring.

Note: sufficiently accurate results can normally be achieved by modelling only a proportion of the actual physical units in a given ring (e.g. 40 units per ring is often acceptable for a medium-span arch); this often reduces run times considerably, with only moderate loss in accuracy. However, it should be noted that collapse load predictions obtained using this strategy may be slightly non-conservative.

13.3.4 Ring thickness

Enter the ring thickness, in mm.

User-defined rings also have the option to have a non-uniform thicknesses. By unchecking the **Assume uniform ring thickness** box, a further tab is added to the coordinates panel in which

extrados ring points can also be added (using the same method as for the intrados). The ability to turn off the **Bed joints normal to intrados** option also becomes available.

13.3.5 Inserting a span

LimitState:RING allows the user to insert new spans into the project without having to build the whole bridge again from scratch. Simply select the tab of a span adjacent to the place where a new one is to be inserted and click the **Insert span** button. LimitState:RING will ask the user to decide whether the new span is to be inserted to the left or right of this position and, after a choice has been made, a new span and new pier will be added. LimitState:RING will also automatically renumber all objects to accommodate these changes. The inserted span will assume the geometry of the original selection. However the material properties will retain the default values.

Note:

1. *After inserting a new span the dialog remains on the original tab - not that corresponding to the span that has just been inserted.*

2. *It is prudent to check the properties of the inserted span*

13.3.6 Deleting a span

LimitState:RING also allows the deletion of spans. After selecting the span to be deleted in the geometry dialog (by clicking the appropriate tab), and clicking the **Delete span** button, LimitState:RING will either choose the associated pier to delete (if the span is connected to an abutment) or ask the user to decide (where the span is not connected to an abutment). Re-numbering of the remaining objects will be done automatically.

13.4 Piers

To edit the geometry of a pier, simply click on the relevant tab to display the dialog in Figure 13.4.

© *LimitState Ltd*

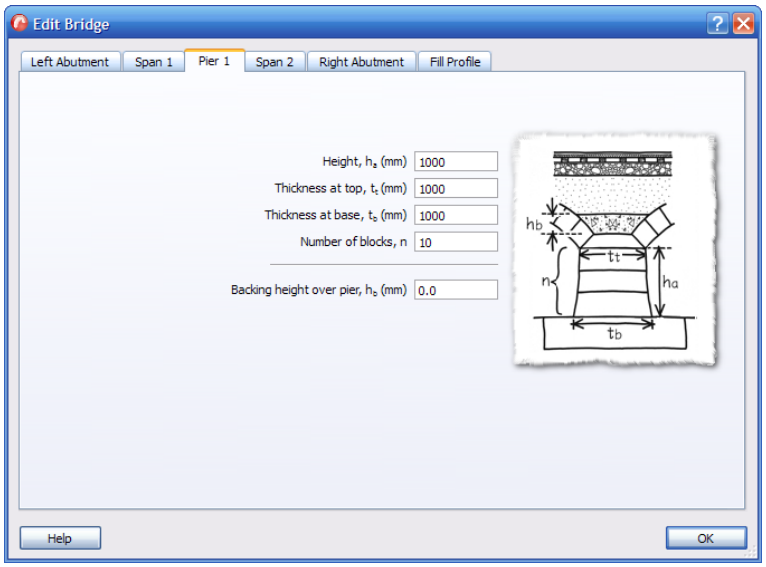


Figure 13.4: Geometry - Pier Properties

13.4.1 Default pier model

By default, LimitState:RING presumes that all piers are constructed similarly, in that they are each 1000mm high by 1000mm wide (at both the top and the base) and that they are all constructed from ten blocks with no backing above them.

13.4.2 Modelling piers explicitly

To override the default pier model, simply enter new values in the relevant boxes of each pier. Aspects that can be modified include:

- Height
- Thickness at top
- Thickness at base
- Number of blocks
- Backing height (see Section 5.8.4 for more details)

13.5 Fill profile

To edit the upper and lower profiles of the surface layer, simply click on the relevant tab to display the dialog in Figure 13.5.

Taking the left springing position of the first arch as the origin, enter the *x* and *y* co-ordinates at a point on the lower edge of the surface fill (highway) or ballast (railway) and give the depth at that point.

x the horizontal distance from the left springing point of the first arch (mm)

y the vertical distance from the level of the left springing to the base of the surface fill (mm)

Surface fill / **ballast depth** the depth of surface fill or ballast from the top surface to point (x,y) (mm)

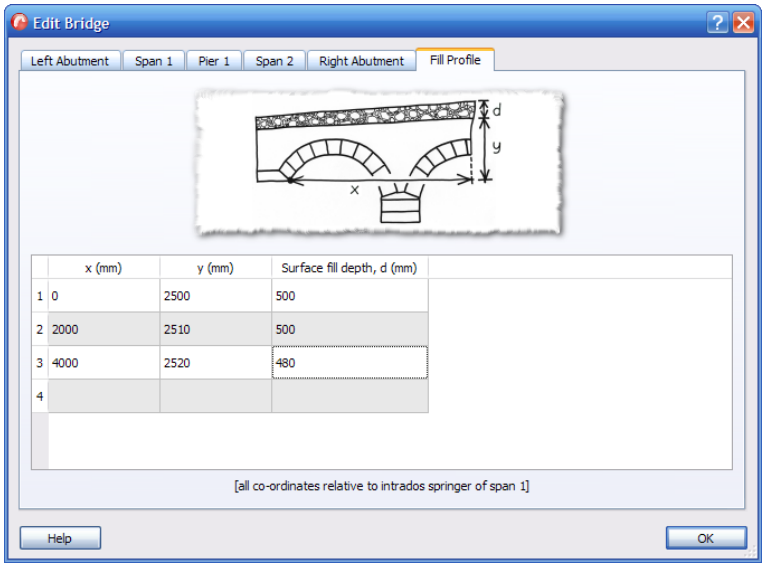


Figure 13.5: Geometry - Surface Fill

Note:

1. *The x and y distances are always relative to the left intrados springing of Span 1*
2. *User-defined fill profiles are formed from a series of straight lines that intersect the points specified.*

© LimitState Ltd

Chapter 14

Material properties

With the exception of backing (see Section X), the material properties can be found and modified within the **Materials** dialog (click **Tools** >**Materials**). Alternatively the command may be accessed via the toolbar, and the keyboard shortcut for the command is **Ctrl+4** or by clicking the **Materials Dialog** icon on the **Properties** toolbar.. This will open the dialog shown in Figure 14.1.

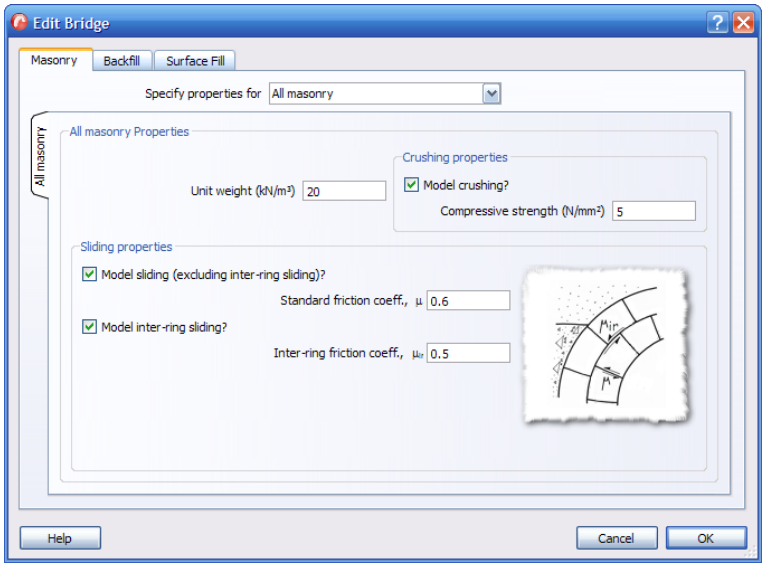


Figure 14.1: Material properties dialog box

14.1 Masonry

14.1.1 'Specify properties for'

The **Specify properties for:** drop-down box on the **Masonry** tab allows the user to change the properties of different masonry features within the project:

All masonry

This is the default option and changes made here will affect the material properties of all masonry objects. The tab for all masonry remains on show, whichever option is chosen in the drop-down. This allows the user to get an overview of the project and make wholesale changes if this is required.

Spans vs. piers / abutments

Selecting this option generates three further tabs in the properties box:

1. All spans - alterations made here affect all spans in the project
2. All piers - alterations made here affect all piers in the project
3. All skewbacks - alterations made here affect all skewbacks in the project

All bridge parts

Selecting this option generates tabs in the properties box which correspond to each individual part of the bridge. In addition, tabs for **All masonry**, **All spans**, **All piers** and **All skewbacks** are also generated, so that the user can make changes to all similar features in the project without the need to alter each individual tab.

14.1.2 Unit weight

Here, it is possible to specify the unit weight (in kN/m^3) of the masonry features under consideration.

14.1.3 Model crushing

By default, crushing is modelled, with the default crushing strength taken as 5 N/mm^2. To model the blocks as being rigid, uncheck the **Model crushing?** option and enter a compressive strength for the material under consideration.

Note:

1. *When crushing strength is included in the analysis the problem becomes non-linear, and several iterations will normally be required before a converged solution is obtained. This means that the computational effort required to obtain a solution is increased. LimitState:RING uses a highly robust solution scheme, obviating the need to specify convergence tolerance etc (required with LimitState:RING 1.x).*

© LimitState Ltd

2. *In LimitState:RING a moment vs. normal force failure envelope which assumes ductile response of the masonry is assumed; it is assumed that a given hinge in the failure mechanism forms at the edge of a rectangular stress block.*

3. *If crushing strength is included in the analysis LimitState:RING will find a solution (though see note below) even if the structure had been found to be 'geometrically locked' when crushing strength was assumed infinite (unlike LimitState:RING 1.x, which will provide no solution in such circumstances)*

4. *When very low crushing strengths are specified it may be impossible to obtain a solution.*

14.1.4 Sliding properties

LimitState:RING models potential sliding between blocks, both within piers and rings and between adjacent rings, though this feature can be switched off if required.

Inter-block sliding

To model sliding between all blocks (except between adjacent arch rings), ensure that the **Model sliding** option is checked and enter a value for the standard coefficient of friction.

Note:

1. *LimitState:RING 3.2.a models friction by assuming that sliding between adjacent blocks is accompanied by separation (so-called 'dilatant' friction, or 'plastic shearing'). For most arch problems this assumption has been found not to affect the computed load factor significantly. However, it should be borne in mind that strictly speaking the computed factor is an 'upper bound' on the exact load factor (though this 'upper bound' often coincides with the exact value). Refer to Gilbert & Melbourne 1994 for more details*

Inter-ring sliding

To model sliding between rings, ensure that the **Model inter-ring sliding** option is checked and enter a value for the coefficient of friction between adjacent rings.

Note:

1. *For problems involving several rings it has been found that the modelling of friction by assuming that separation accompanies relative sliding between rings often leads to reasonable estimates of bridge strength (e.g. computed strengths often found to agree well with Bolton arch and arch bridge strengths - refer to Melbourne & Gilbert 1995). However, it should be borne in mind that strictly speaking the computed factor is an 'upper bound' on the exact load factor.*

© LimitState Ltd

14.2 Backfill - Standard properties

To edit the backfill properties click on the **Backfill** tab of the **Materials** dialog as shown in Figure 14.2.

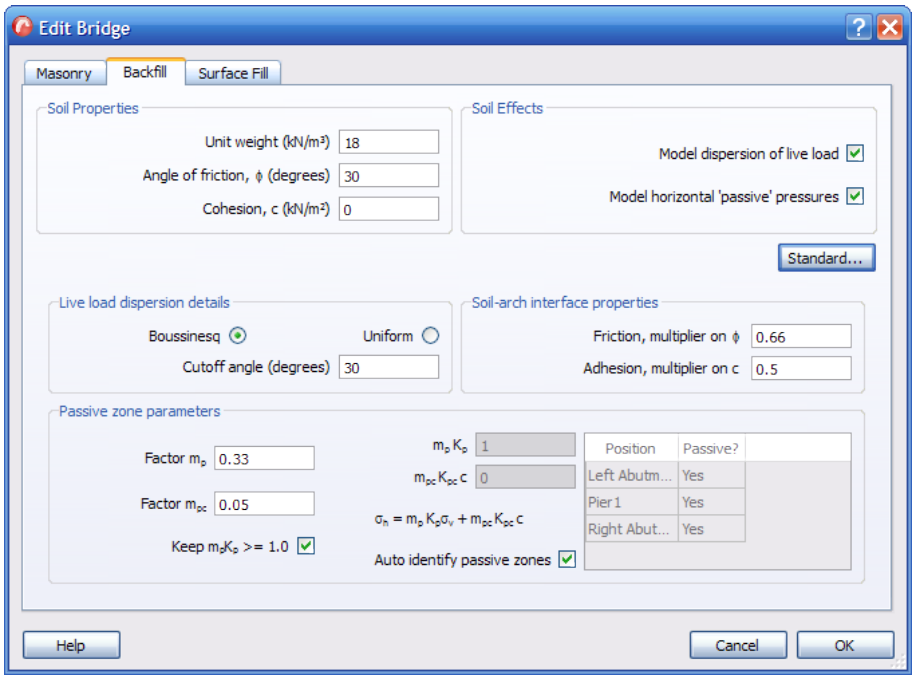


Figure 14.2: Backfill properties tab

14.2.1 Soil Properties

Unit weight

Specify the unit weight (in kN/m^3) of the backfill material.

*Note: If you wish to model an arch without fill, specify a zero value for the backfill unit weight (also uncheck **Model dispersion of live load**).*

Angle of friction, ϕ

Specify the angle of friction ϕ (in degrees) of the backfill material.

Cohesion, c

Specify the cohesion c (in kN/m^2) of the backfill material.

Note: These soil properties are used by the selected backfill numerical model. In the current version of LimitState:RING, the 'standard' model is provided which is backwards compatible with RING 1.5 and provides good correlation with published experimental data.*

(*There is a small difference between how RING 1.5 and LimitState:RING 3.2.a models Boussinesq load spreading, which may have a minor effect on collapse load calculations for a small subset of loading scenarios. The import facility in LimitState:RING will automatically convert RING 1.5 backfill settings to the correct equivalent values so that identical pressure distributions are modelled in LimitState:RING 3.2.a . Further details of this process are given in Appendix B.6.

14.2.2 Soil Effects

Model dispersion of live load

Check this box to specify that the backfill model should include dispersion of the live load. When unchecked, loading on the bridge structure will occur over a longitudinal distance equal to that at the base of the surface layer. Where the effective bridge width is set to **Auto-compute** (see Section 12.1.2) dispersion through the backfill in the transverse direction is controlled via the **Effective bridge width parameters** dialog.

Model horizontal 'passive' pressures

Check this box to specify that the backfill model should include modelling of the **passive pressures** (see Section 5.8) arising when a the arch moves into the backfill.

14.3 Backfill - Advanced properties

14.3.1 Live load dispersion details

Boussinesq

Select this option to specify that the magnitude of the pressure exerted on the back of the arch is to be calculated according to the Boussinesq equation. This is the default option.

Normally a cutoff angle will be specified (default: 30 degrees) because experiments have shown that when arch movements become large (e.g. at failure) a cone of soil under the applied load tends to punch through. When a limiting distribution angle is specified the magnitudes of the pressures calculated using the Boussinesq equation are scaled up so that that the integral of the vertical pressures acting on a length of arch equals the magnitude of the applied force. The length of arch assumed to be subject to vertical loading pressures is indicated Figure 14.3.

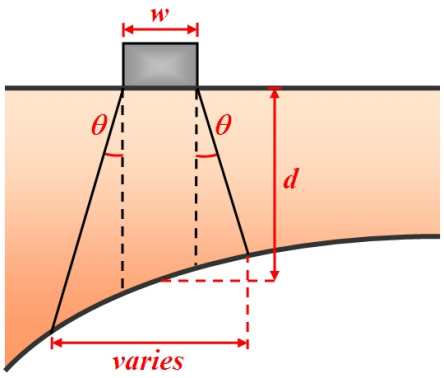


Figure 14.3: Cutoffs used with Boussinesq load dispersion model

Uniform

Select this option to specify that the magnitude of the pressure exerted on the back of the arch is to be constant. The length of arch assumed to be subject to vertical loading pressures is controlled by the specified cutoff angle as indicated Figure 14.4.

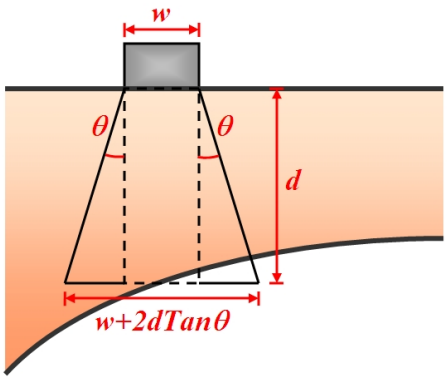


Figure 14.4: Cutoffs used with uniform load dispersion model

Note:

1. *Use the **Uniform** option to comply with the requirements of some assessment codes. Note, however, that the Boussinesq option is likely to provide a more realistic representation of the actual distribution of fill pressures as has been indicated by numerical and experimental studies.*

© LimitState Ltd

14.3.2 Soil-arch interface properties

Friction, multiplier on ϕ

Specify the multiplier on the soil angle of friction ϕ that gives the soil-arch interface angle of friction δ.

Adhesion, multiplier on c

Specify the multiplier on the soil cohesion c that gives the soil-arch interface adhesion a. This multiplier will depend on the nature of the arch extrados masonry.

Note:

1. *The friction multiplier on ϕ will depend on the nature of the arch extrados masonry. However in many geotechnical codes for retaining walls, δ is a function of the critical state angle of shearing resistance, which will not necessarily be the value of ϕ entered for the backfill.*
2. *In the LimitState:RING 'standard' backfill model, the soil-arch interface properties are used to calculate an upper limit on the magnitude of the horizontal backfill pressures that can be applied to a given masonry block without causing the strip of backfill on the block to slide. The specified interface properties are only used for the above purpose, and are not used to calculate frictional energy dissipation at the backfill/arch barrel interface (e.g. for use in the work equation).*
3. *The check on the limiting horizontal backfill pressures that can be applied is overridden when user-defined horizontal forces / pressures are specified.*

14.3.3 Passive zone parameters

In the LimitState:RING 'standard' backfill model, soil pressures in the passive zone are determined using modified lateral earth pressure theory. This idealization is discussed further in Section 5.8.3. The horizontal stress σ_h is given by:

$$\sigma_h = m_p K_p \gamma z + m_{pc} K_{pc} c \tag{14.1}$$

where γ is the unit weight of the backfill and z is the depth of fill at the point where the pressure is being calculated. m_p and m_{pc} are user defined pressure modification factors.

Factor m_p

Specify the factor m_p for determining the resultant lateral earth pressure arising from backfill self weight as defined in equation 14.1. The resultant value of $m_p K_p$ is given in the adjacent

© LimitState Ltd

box.

Factor $\mathbf{m}_{pc}$

Specify the factor m_{pc} for determining the resultant lateral earth pressure arising from the backfill cohesive strength as defined in equation 14.1. The resultant value of $m_{pc}K_{pc}c$ is given in the adjacent box.

Note:

1. *Small changes to the specified passive zone parameters can lead to large changes in the computed collapse load. Hence care must be exercised when selecting these values.*
2. *The horizontal backfill stresses defined above may be reduced by the program if these are sufficiently high to cause relative sliding between the backfill and the arch barrel. (See Section 14.3.2.)*
3. *The values of the backfill pressures calculated here can subsequently be modified if required. See the* ***Viewing and modifying attributes*** *section.*
4. *To specify that horizontal backfill pressures of constant magnitude H will be mobilised when the arch sways into the backfill, set the* ***Factor*** m_p *to zero and the* ***Factor*** m_{pc} *to a suitable value to generate a resultant $m_{pc}K_{pc}c$ equal to H.*
5. *These equations are given assuming dry conditions. To model conditions where the arch is flooded to above fill level, enter buoyant unit weights for both masonry and backfill.*

Keep $\mathbf{m_pK_p} >= \mathbf{1.0}$

Checking this box ensures m_pK_p is always greater than or equal to 1.0. The resultant value of m_pK_p can fall below 1.0 for low values of ϕ and m_p, which can be unrealistic. See Appendix B for further discussion of this point.

Auto identify passive zones

Checking this box causes uniaxial fill elements to be included in the analysis. These elements are positioned horizontally in the spandrel void area(s). Elements are initially placed in contact with every block in the arch extrados.

The elements exhibit the following characteristics:

- The elements are constrained to either stay the same length or to compress.
- The elements exhibit a rigid-plastic response in compression (i.e. they compress at a constant force. This force is equal in magnitude to the specified fill pressure multiplied by the vertical height of the extrados face of an arch block)

© LimitState Ltd

- For an element positioned above a rigid abutment, the end of the element remote from an arch block is assumed to be fixed in position.

- For an element positioned above an abutment block, the end of the element remote from an arch block is assumed to be fixed to a vertical line drawn from up from the centroid of the top block in the abutment. This means that the element will only compress if there is a relative closing movement between the backfill above the abutment block and the arch block to which the element is attached (in other words, no horizontal backfill pressures need be mobilised if blocks in an arch move, say, to the left provided the skewback on top of the abutment also slides to the left). This approach effectively assumes that there is no additional backfill, say, to the left of the abutment block; this is true for the case of an arch span adjacent to a beam span.

Note:

1. *In this release uniaxial fill elements cannot be fixed to the sides of pier/abutment blocks.*

Unchecking the 'Auto identify passive zones' box causes the fill elements to act both in the 'passive' and 'active' senses, i.e. applying pressure to the arch whether this moves towards or away from the fill. The user can then manually define in which areas of the bridge the pressures will be applied. This is achieved by editing values in the passive restraint table (see Figure 14.5). Select 'yes' to include horizontal pressures and 'no' to not include horizontal pressures (vertical pressures will still be modelled).

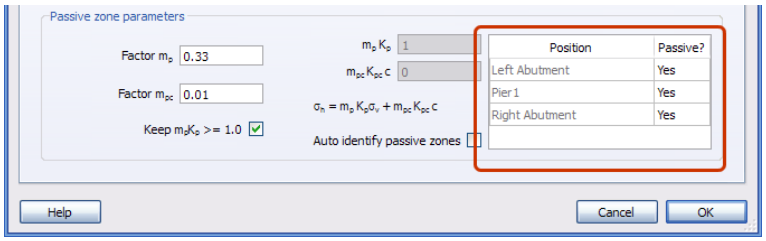


Figure 14.5: Backfill - Passive restraint table

For example, for a single span arch which is to be loaded to the left of the crown, fill pressures should be specified only to act on the right-hand-side (RHS) of the arch. However, this approach is likely to be problematic in many cases (e.g. for multi-spans, multiple load case analyses, deep arches etc). Fortunately for the user if pressures are not mobilised in the correct (i.e. 'passive) sense, the computed load factor will be a lower bound on the exact load factor.

Note: Some additional computational effort is required when fill elements are included in the analysis. Thus in certain situations (e.g. when it is obvious in advance that fill pressures will be mobilized in a given zone) there may be justification for switching off the automatic-detection of fill pressures.

14.4 Backing properties

Bridge backing is modelled in LimitState:RING as a special implementation of the backfill model (see Section 5.8). The assumed default properties are as follows:

Unit weight Equal to the unit weight of the backfill (this is likely to be a conservative estimate of the true unit weight).

Compressive strength 5MPa default (although this can be overridden if desired - see below).

Tensile strength 0MPa (i.e. tensile forces / active pressures are not permitted).

The compressive resistance offered by the backing can be modified in the following way:

- Select the **Block(s)** that correspond to the backing elements that are to be modified.
- In the **Property Editor**, set ***Fill force (H): Max*** to the desired force. Note that the value should be:

$$\text{Force} = \sigma_{h,backing} \times \text{ vertical projected block area}$$

where $\sigma_{h,backing}$ is the allowable horizontal stress for the backing and the vertical projected block area assumes a 1m block depth 'into the page'.

Where backing exists over a pier, there exists the possibility for two backing elements to be associated with a block (one from each of the two associated spans). In such cases, if there is a discrepancy in the allowable force that the backing is permitted to assume, the lower (more conservative) value will be chosen.

Should the limiting force of a backing element be reached for any particular analysis, the element will turn green in the viewer (in contrast to active backfill elements, which turn blue).

14.5 Surface fill

The last tab in the materials dialog concerns the properties of the surface fill material (or ballast material in the case of railways) . This is assumed to be uniform throughout the model and, therefore, is given a uniform dispersion model.

14.5.1 Unit weight

Enter a unit weight (in kN/m^3) for the surface fill / ballast material.

© *LimitState Ltd*

14.5.2 Angle of dispersion

Enter an angle of load dispersion for the surface fill / ballast material. This will be used to calculate the loading pressures on the underlying structure using a uniform model.

© *LimitState Ltd*

© *LimitState Ltd*

Chapter 15

Loading

In LimitState:RING a database of loading vehicles can be set up and then used in load cases. Multiple load cases can rapidly be set up by copying and repositioning an existing load case at regular intervals across a bridge.

On the **Tools** menu, click **Loading** to obtain the **Loading** dialog shown in Figure 15.1. Alternatively, click the **Loading** icon, .

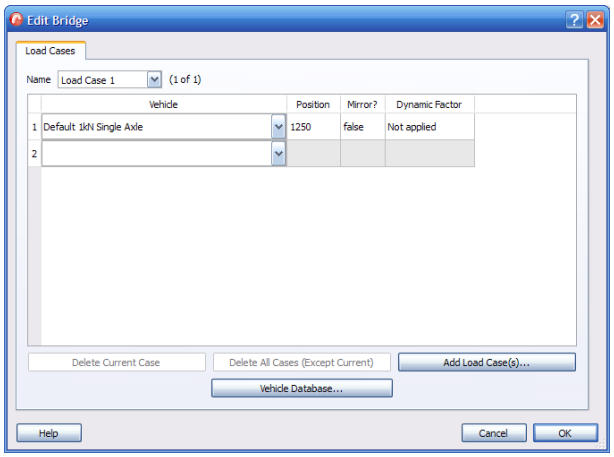


Figure 15.1: Loading dialog

15.1 Mirrored vehicles

Many loading vehicles possess an irregular spacing of axles and it can therefore be necessary to consider the path across the bridge in both directions - from left to right and from right to left.

To facilitate this, LimitState:RING allows the user to 'mirror' a vehicle when setting up the load cases. To do this, simply select *true* in the ***Mirror*** dropdown box of the **Loading Dialog**. If there are multiple load cases in the problem, a secondary dialog will appear asking whether you wish to change the mirror property for all cases, or just the one in question.

15.2 Adding a vehicle to the project

In order to add loading to a bridge, vehicles must be either:

1. Imported into the project from the existing database
2. Imported from file
3. Newly defined using LimitState:RING

In all of these cases, the **Vehicle Database...** button must be clicked to obtain the **Vehicle Database Dialog** shown in Figure 15.2.

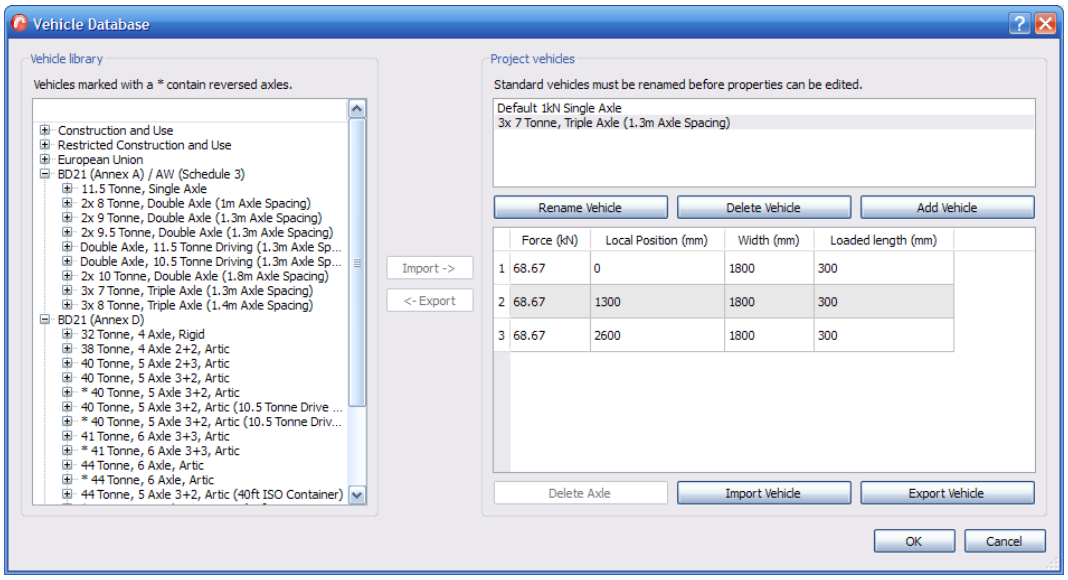


Figure 15.2: Vehicle database dialog box

15.2.1 Importing existing vehicles

The current vehicles in the database are categorized by family. To import one of these for use in the project:

1. Choose a vehicle from the list on the left (by expanding the tree, you will be able to view the properties for each one before importing).
2. Import the vehicle into the project by clicking on the vehicle and using the **Import** arrow button (the vehicle details are displayed in the right-hand window).
3. Click **OK**.

The vehicle is now available for use in any load case, although it will not have been specifically allocated to any one.

You may also export vehicles (e.g. user-defined) from the project by selecting them and clicking the **Export** arrow button.

15.2.2 Defining a new vehicle using properties saved in a file

Click on **Import vehicle** to import details of a vehicle previously saved in a tab separated variable text (.txt) file. This type of file can easily be exported from a spreadsheet, or can be created using a text editor such as Windows Notepad. The Notepad text file shown in Figure 15.3 would generate the same library entry as was entered manually in the dialog above:

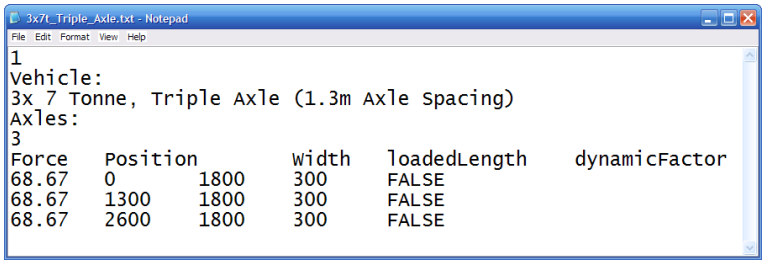


Figure 15.3: Defining a new vehicle from file

The first row of the file specifies whether the vehicle is editable or not (enter 0 for an editable vehicle or 1 for a non-editable vehicle).

The second and third rows of the file define the name of the vehicle.

The fourth and fifth rows of the file define the number of axles.

The sixth row specifies the labels for the vehicle data (this should be copied exactly).

The remaining rows in the file are the specific data relating to each axle:

Force The force (or load) imparted by the axle in kN.

Position The position (mm) of the axle.

Width The width of the axle.

Loaded length The longitudinal length (mm) of wheel in contact with the surface (rail or road).

Dynamic factor Specifies whether the axle is subject to dynamic partial factors, as outlined in Section 16.

Using a spreadsheet the same data could have been entered as shown in Figure 15.4, and then saved as a 'Text (Tab Delimited)' (.txt) file prior to being read in by LimitState:RING:

	A	B	C	D	E
1	1				
2	Vehicle:				
3	3x 7 Tonne, Triple Axle (1.3m Axle Spacing)				
4	Axles:				
5	3				
6	Force	Position	Width	loadedLength	dynamicFactor
7	68.67	0	1800	300	FALSE
8	68.67	1300	1800	300	FALSE
9	68.67	2600	1800	300	FALSE

Figure 15.4: Defining a new vehicle using a spreadsheet

15.2.3 Defining a new vehicle within LimitState:RING

To define a completely new vehicle within the software, click on the **Add vehicle** button and enter the name of the vehicle to be added. Details of the vehicle can then be added to the **Vehicle Database** as shown in Figure 15.5. Note that, if defining a vehicle in this way, the **Dynamic** axle factor is specified in the **Loading** dialog, and not within the **Vehicle Database** itself.

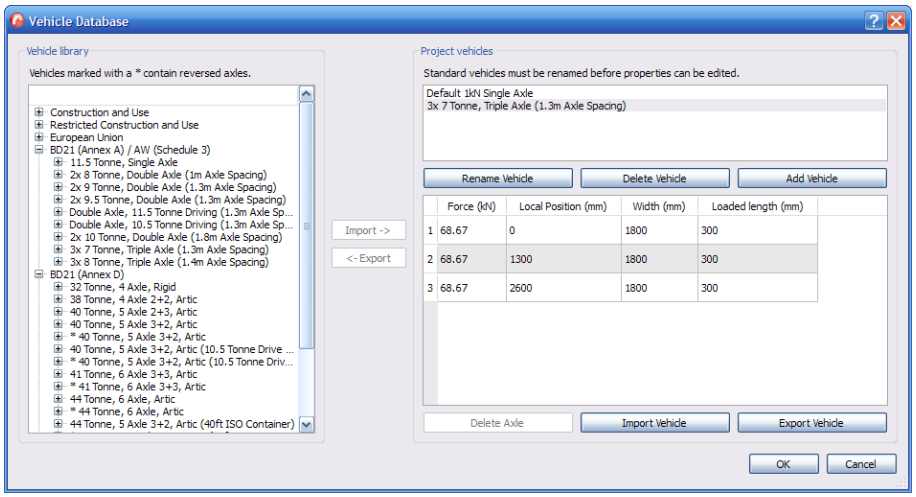


Figure 15.5: Defining a new vehicle within LimitState:RING

15.2.4 Editing vehicle properties

The name, position and force exerted by the axles of a vehicle can all be changed by modifying the properties in the right-hand window of the **Vehicle Database**.

If several vehicles are being used in the project, be sure to use the drop down box to select the correct one.

15.2.5 Renaming a vehicle

To rename a vehicle, click on the **Rename Vehicle** button and enter the new text.

15.2.6 Deleting a vehicle

To delete a vehicle, click on the **Delete Vehicle** button. Note that there must be at least one vehicle present at all times.

15.2.7 Exporting a vehicle to a file

You may wish to save a customized vehicle for use in later LimitState:RING projects. To do this, click on **Export vehicle** to export details of a vehicle to a tab separated text (.txt) file. The contents of this file may subsequently be imported back into LimitState:RING, or viewed in a spreadsheet.

15.3 Adding a vehicle to a load case

Once all of the necessary vehicles have been imported into the project they can be allocated to load cases. To do this, simply return to the **Loading** dialog, select the relevant load case (if there is more than one) and click the appropriate box in the **Vehicle** column. This will bring up a drop-down menu containing all of the available vehicles as shown in Figure 15.6.

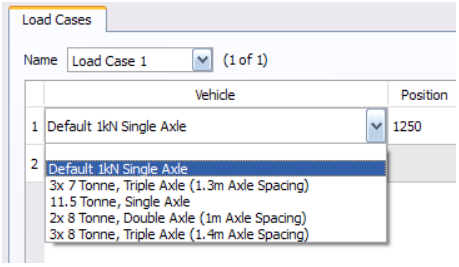


Figure 15.6: Vehicles available in the current project

The relevant vehicle can now be selected. Enter its position on the bridge.

Note: It is usually sensible to position the first axle in a vehicle at a position of 0mm. Axles in the vehicle to the right of this axle are then indicated by positive distances entered in the table. In the case of an unsymmetrical bridge crossed by an unsymmetrical vehicle, a 'mirror' can be applied to the vehicle so that it can cross the bridges in both directions.

Several vehicles can be used to form a load case as shown in Figure 15.7.

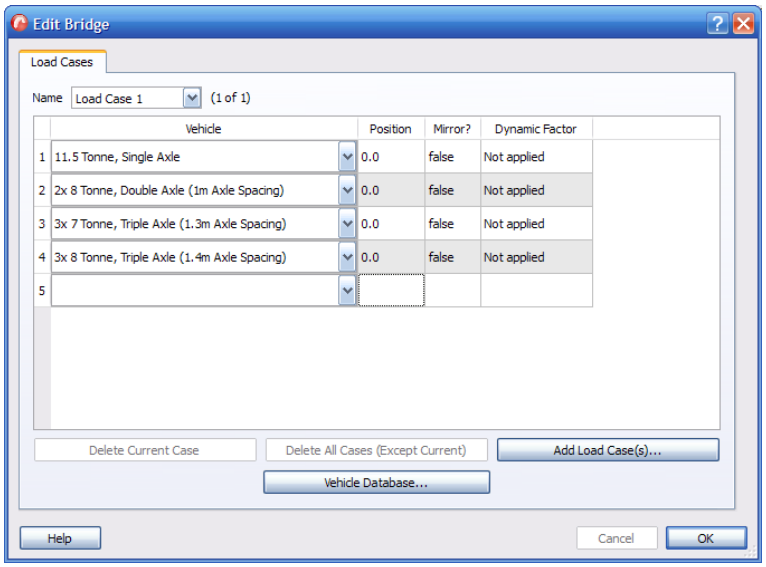


Figure 15.7: Multiple vehicles in a load case

Note:

1. *The vehicle positions specified are relative to the assumed bridge datum point, which is the left hand (intrados) springing of span 1.*

2. *When multiple vehicles are specified in a single load case they are assumed to be acting together. If you wish to analyse a bridge for several different vehicle types then separate models should be created, one for each.*

15.3.1 Assigning 'dynamic' factors

Codes of practice sometimes require the consideration of 'dynamic' or 'impact' loading when assessing the capacity of a bridge structure. Often this is implemented via the application of a 'dynamic load factor' to one or more axles (maybe the most heavily loaded) of the loading vehicle.

LimitState:RING now allows dynamic axle loading to be specified on a per-load case basis. When used in an analysis, this axle will then be multiplied by whatever 'Dynamic' partial factor is set in the **Partial Factors Dialog** (see Section 16).

15.4 Defining new load cases

15.4.1 Adding load cases

To add further load cases click on **Add Load Case(s)** to obtain the dialog shown in Figure 15.8.

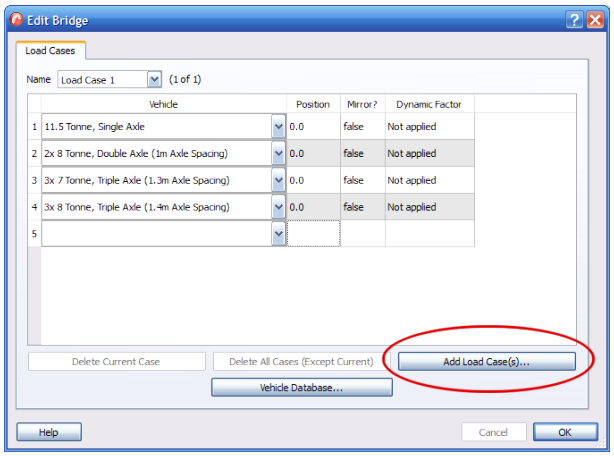


Figure 15.8: Adding a new load case

Copy an existing load case

In most situations, it is easiest to create one or more new load cases by incrementing the positions of an existing load case. In this case you should choose to **Copy an existing load case**. You may now enter the existing load case number, the total number of copies to be made and the spacing (offset) between each of the new load cases to be generated.

Creating an empty load case

A single *new* load case can be set up manually by clicking **Create an empty load case** in the **Add new load case(s)** dialog. This can then be edited as normal.

15.4.2 Deleting a load case

Deleting a single case

In the ***Name*** dropdown menu of the **Loading dialog**, select the load case to be deleted (e.g. Load Case 1 shown in Figure 15.9) and then click the **Delete Current Case** button.

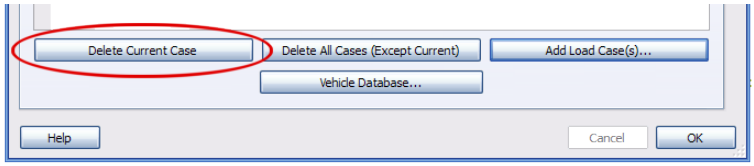


Figure 15.9: Deleting a single load case

Note: There must always be one load case in the analysis.

Deleting all but one case

In the ***Name*** dropdown menu of the **Loading dialog**, select the load case to be retained (e.g. Load Case 3 shown in Figure 15.10) and then click the **Delete All Cases (Except Current)** button. All of the cases except that which is selected will then be removed from the problem.

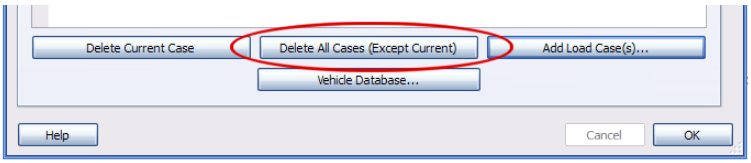


Figure 15.10: Deleting all but the current load case

Note: There must always be one load case in the analysis.

15.5 Moving vehicles across a bridge

A common goal when using LimitState:RING is to identify the critical position of a loading vehicle on a bridge (i.e. that position that results in the lowest adequacy factor). To achieve this the user can follow one of two methods:

15.5.1 Method 1 - Multiple load cases

A vehicle can be moved incrementally across a bridge by setting up a series of **load cases** (see Section 15.4.1). In this way the user can quickly identify the critical load position:

1. Determine the range of load positions that will be required to adequately cover the bridge. You can do this by dragging the vehicle in the **Viewer window** to the left and right sides of the model and noting the distances at which the loading is just touching the spans.
2. Open the **Loading dialog**
3. Ensure that the vehicle you wish to move across the bridge is selected in the dropdown menu.
4. In the **Position** field for the vehicle, enter the far left distance as noted in Step 1.
5. Click the **Add Load Case** button.
6. Select **Copy an existing load case.**
7. Select the target load case.
8. Specify the number of loads that will be needed to adequately cover the whole bridge with a reasonable spacing. For example, for a bridge with loading from -1000mm to 6000mm, 40 additional load cases would require a spacing of 175mm. The exact number and spacing of the cases is left to the judgement of the user.

9. Click **OK** - the new load cases will be generated.

Now when the problem is solved, all loading cases will be considered. Note that If you need to model different vehicles, you should setup different models, since the necessary load case spacing will differ between vehicles.

15.5.2 Method 2 - Drag and solve

An alternative method of finding the critical load position is to enable automatic recalculation whenever the load vehicle is moved in the **Viewer pane**.

To enable this feature, go to the **Tools** >**Preferences** dialog. Select the option to **Solve automatically after dragging a vehicle** and close the dialog.

Ensuring that there is only a single load case in the problem, click and hold the left mouse button over the loading vehicle in the **Viewer pane**. Drag the vehicle to a new position using the mouse and release the button. The problem will solve automatically with the load at the position you have specified. Using a process of trial and error, the critical location can be established.

15.6 Moving a load in a multiple load case problem

Load case positions can be altered by specifying a new value in the ***Position*** field of the **Loading Dialog**. If a position is changed when multiple load cases are present, the user will be asked whether to move all cases by the same increment, or just to move the current case.

15.7 Viewing load cases

To view a particular load case (and, if an analysis has just been performed, also the associated collapse load factor) select this using the **Up**/**Down arrows** in the **Load case spinbox** (see Section 20.4.1).

© *LimitState Ltd*

© LimitState Ltd

Chapter 16

Partial factors

LimitState:RING has been designed to be 'code agnostic'. That is, the assignment of partial factors on load, material strengths etc. is left entirely up to the user.

The available partial factors are detailed in Table 16.1, all are set to 1.0 by default but can be overridden:

Partial factor name	**Symbol**	***Notes***
Masonry unit weight	$\gamma_{f,m}$	*Load factor - applied to permanent load from masonry*
Fill unit weight	$\gamma_{f,f}$	*Load factor - applied to permanent load from backfill*
Surface fill / ballast unit weight	$\gamma_{f,sf}$	*Load factor - applied to permanent load from surface fill / ballast*
Track load	$\gamma_{f,t}$	*Load factor - applied to permanent load from track*
Axle load	$\gamma_{f,l}$	*Load factor - applied to variable load from vehicle axles*
Dynamic	$\gamma_{f,dyn}$	*Load (impact) factor - applied to axles where **'Dynamic Factor = true'** (see Section 15.3.1) has been set*
Masonry strength	$\gamma_{m,ms}$	*Material factor - applied to masonry crushing strength*
Masonry friction	$\gamma_{m,mf}$	*Material factor - applied to masonry friction coefficients*

Table 16.1: Default partial factors

© LimitState Ltd

Chapter 17

Reinforcement

17.1 Properties

LimitState:RING allows the user to include reinforcement in the model, meaning that the the software can also be used to assess a range of reinforced arch bridges (refer to Section 5.5 for advice on the assumptions made with respect to the reinforcement model).

Up to two layers of reinforcement can be added at at 90° across any **Contact** by specifying the distance measured from an end (*A* or *B*) and the limiting tensile and compressive forces[1] (i.e select radial contacts to assign circumferential reinforcement and vice versa). For example, see Figure 17.1, where circumferential reinforcement has been added to the bridge model:

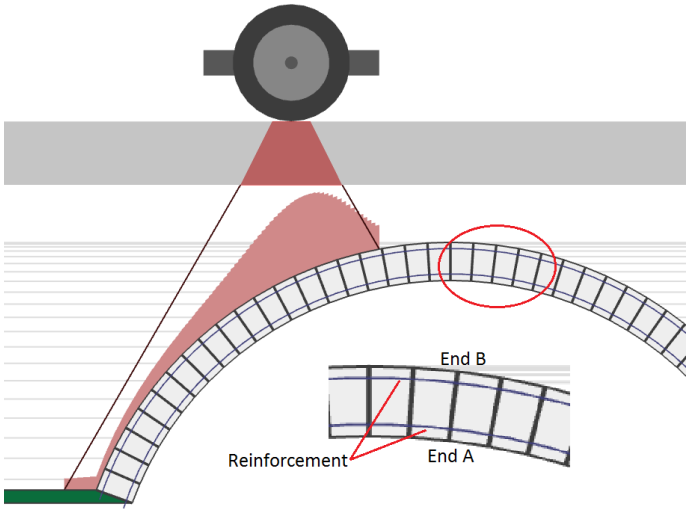


Figure 17.1: Reinforcement added to a bridge model

In a project that includes reinforcement (see Section 17.2) the properties of the reinforcement

[1] In a pre-release version of the software only a single limiting force value could be specified. When files saved with the earlier version are loaded in LimitState:RING, both the tensile and compressive limiting forces will be set to this single limiting force value, though can be changed subsequently.

across each **Contact** are displayed in the **Property Editor**:

Property	Details
Reinforcement depth (A)	The depth (mm) to the reinforcement from contact endpoint A (intrados if radial). Note that this must be a positive value, equal to less than half the contact length.
Reinforcement depth (B)	The depth (mm) to reinforcement from contact endpoint B (extrados if radial). This must be a positive value, equal to less than half the contact length.
Reinforcement max. force C (A)	The limiting compressive force (kN per metre bridge width) in the reinforcement nearest endpoint A.
Reinforcement max. force T (A)	The limiting tensile force (kN per metre bridge width) in the reinforcement nearest endpoint A.
Reinforcement max. force C (B)	The limiting compressive force (kN per metre bridge width) in the reinforcement nearest endpoint B.
Reinforcement max. force T (B)	The limiting tensile force (kN per metre bridge width) in the reinforcement nearest endpoint B.
Reinforcement shear capacity	The limiting shear force (kN per metre bridge width) of the reinforced section. (Warning: by default this is set at a high value so as to ensure that shear failure is not initially governing).
Shear force (read only)	The shear force transmitted across the contact surface (kN per metre bridge width)

Table 17.1: Reinforcement fields, as displayed in the **Property Editor**

Note that the Reinforcement force is the limiting force per metre width which can be carried i.e: Reinforcement force = (area of reinforcement per metre width) x (yield stress) x (any applied factors)

17.2 Adding reinforcement to the project

By default it is assumed that a model does not include reinforcement - this is to prevent the **Property Editor** being populated with superfluous fields. The user must therefore specify that reinforcement is to be included. This can be done in one of two ways (see Figure 17.2):

1. Whilst setting-up a model using the **New Bridge Wizard**, tick the '**Bridge includes reinforcement?**' checkbox in the **Project** tab.

2. Whilst editing an existing bridge geometry, by ticking the '**Bridge includes reinforcement?**' checkbox in the **Project details** dialog.

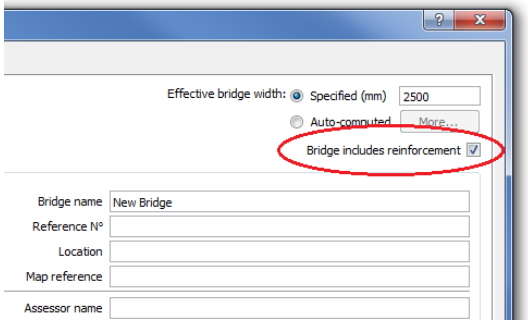


Figure 17.2: Specifying that a project includes reinforcement

Once a model has been defined as containing reinforcement, there is flexibility in the way that the user can add reinforcement to the model; either:

1. Select one or more **Contacts** using the mouse and enter the reinforcement details in the **Property Editor**.

2. Use the **Contact Select Tool**, which can be accessed via the **Select** menu. This tool helps the user to add *circumferential* reinforcement to the entirety of one or more rings or spans by automatically selecting all the *radial* contacts of the chosen part.

   To use the tool, click the name of the bridge part that you wish to select. If more than one bridge part is required, hold down the CTRL key whilst clicking (see Figure 17.3).

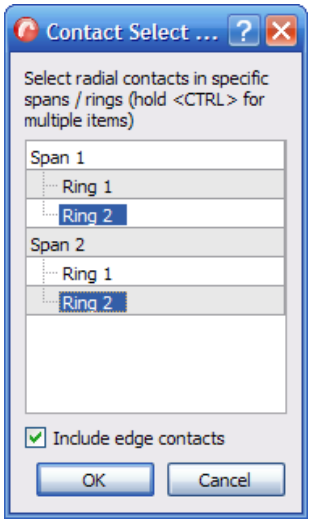


Figure 17.3: Contact Select Tool - the radial contacts in the top ring of both spans of a twin-span bridge will be selected to allow circumferential reinforcement to be added.

   If the end radial contacts in a ring (i.e. those that join the ring to the abutments) also contain reinforcement, ensure the **Include edge contacts** box is ticked (this is selected by default).

   Once the relevant contacts have been selected, click OK. The selected contacts will be highlighted (see Figure 17.4).

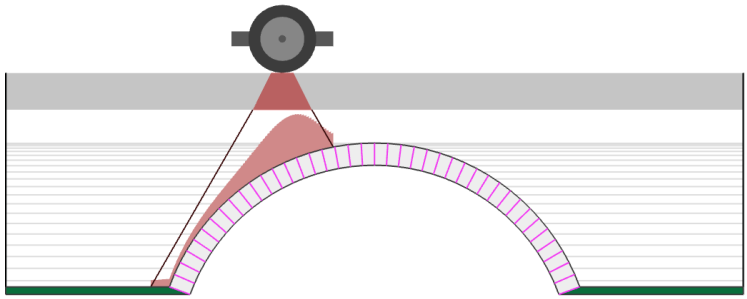


Figure 17.4: Radial contacts selected using **Contact Select Tool**.

Then, in the **Property Editor**, specify the reinforcement position (in millimeters), limiting compressive and tensile forces and the reinforced shear capacity (in kN per metre width). On clicking out of the **Property Editor**, the reinforcement will be displayed on the model (see Figure 17.5)

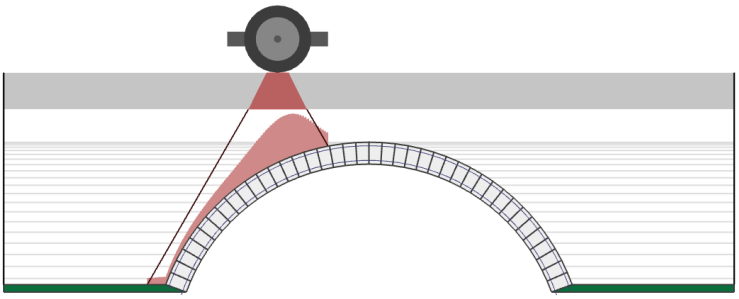


Figure 17.5: Reinforcement added to radial contacts.

© *LimitState Ltd*

Chapter 18

Support movement analysis

18.1 Background

A notable feature in LimitState:RING is the ability to model support movements. This opens up a range of possibilities, for example:

- The likely causes of observed cracks in an existing structure can be investigated by imposing support movements and comparing actual and modelled deformed shapes (e.g. are these consistent with vertical, horizontal or perhaps angular settlement of the base of a pier or abutment?).

- The observed response of a settled bridge can be used to verify the model idealization. A settled bridge can be considered to be of almost the same value as load test to collapse because when a bridge undergoes settlements many of the same modes of resistance are mobilized as when a bridge is subjected to excessive live loading. Therefore it is very useful to try to correlate actual and modelled behaviour (e.g. if it is necessary to include backing in the numerical model in order to replicate the observed mode of response, then this strongly indicates that backing, or very strong fill material, is present in the real structure - and potentially also in similarly constructed structures in the area. This can then be included in subsequent load factor analyses).

- Vehicles can be run across a settled bridge to investigate load paths and to see whether the hinge positions move (if they are predicted to move significantly in the model under traffic, and if secondary stiffening elements such as securely attached spandrel walls are not present in reality, then this might be a cause for concern as continual opening and closing of joints may lead to incremental damage to the structure).

- Once the centering is removed many bridges appear to 'bed down' to a statically determinate (or near statically indeterminate) state. This state can be approximately replicated by moving the supports appropriately. Vehicles can then be introduced and load paths established. If necessary an adequate margin of safety can be ensured by applying a suitable partial factor to the axle loads, ensuring that the structure remains stable.

18.2 Support movement wizard

The easiest way of imposing support movements is to use the Support Movement Wizard, which can be accessed via **Support movements** in the **Tools** menu (Figure 18.1).

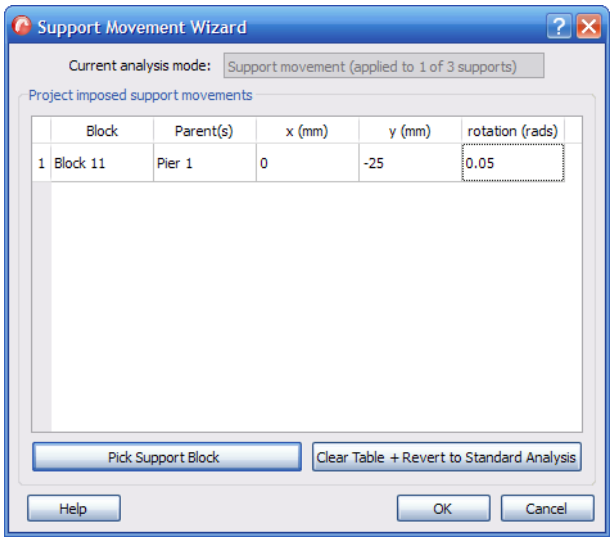


Figure 18.1: Support movement wizard

1. With the **Support Movement Wizard** open, click on **Pick support block to move**.
2. Using the mouse, click on a support (green) block in the modeller window.
3. The wizard will re-appear, now containing details of the chosen support. Enter details of the movement of that block in the table. *Note:*
   - *distances are measured in mm, with positive vertical movement being upwards.*
   - *rotations are measured in radians - anticlockwise about the centroid of the block*
4. Repeat the first two steps until all the required movements have been entered. If, at any time, you wish to return to the wizard without selecting a block, click the **Escape** button.
5. Click **OK** to return to the main window. An analysis may then be carried out as normal, with the prescribed support movement(s) being visible following an analysis.

The result of a support movement analysis is expressed in terms of energy (in Joules). As small-displacement theory is used, computed energy dissipation will vary linearly with the magnitude of the prescribed movement.

Chapter 19

Viewing and modifying attributes

19.1 Using the property editor

The **Property Editor** feature (Figure 19.1) allows the user to quickly read and / or modify the attributes of one or more objects within the current project.

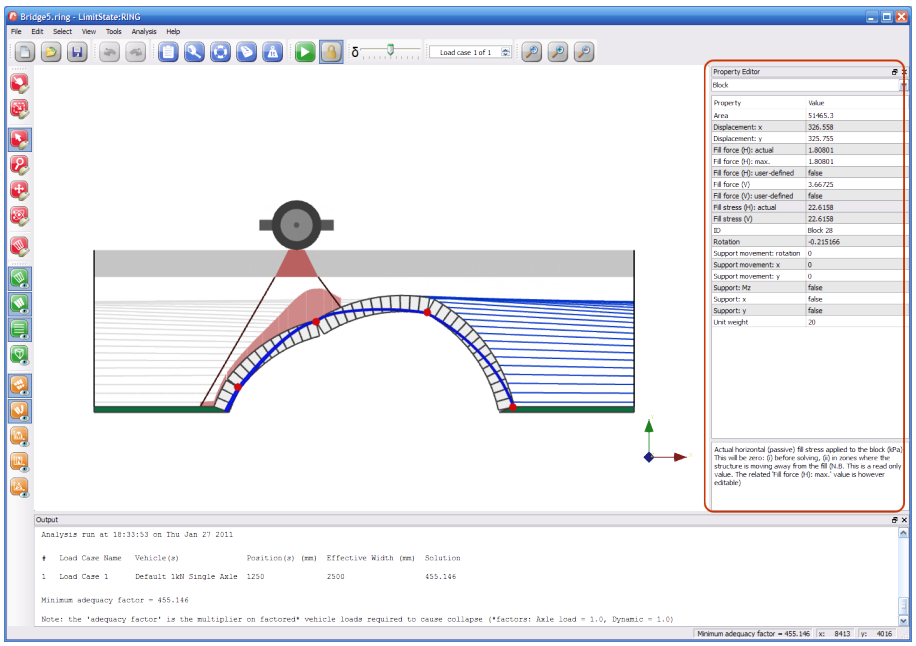


Figure 19.1: The LimitState:RING Property Editor

The **Property Editor** opens by default when LimitState:RING is started, but can be toggled on and off using the **View menu** (see Section 20.3.4).

Some of the functions in LimitState:RING are only accessible via the **Property Editor**. These are described in detail in this section. Other functions and attributes can be accessed and modified elsewhere, but are shown for convenience in the **Property Editor**.

To begin using the **Property Editor**, first select an area of the bridge using the methods described in Chapter 20.

The drop-down menu in the **Property Editor** gives the option of viewing and / or editing the attributes belonging to the different types of object that have been selected.

Table 19.1 lists the objects within the LimitState:RING environment that may be selected, along with sample related functions that can be accessed via the **Property Editor**.

Property Editor option	**Description**	**Unique function(s)**
All	Modify all selected objects	N/A
Fill element(s)	Modify only selected fill elements	Limiting force
Block(s)	Modify only selected blocks	Fill force (H): actual Fill force (H): max. Fill force (H): user-defined Fill force (V) Fill force (V): user-defined Support movement (rotation) Support movement (x) Support movement (y)
Contact(s)	Modify only selected contact surfaces	Enabled Mortar loss (A) and (B) Permit hinges Permit sliding The following are only accessible when reinforcement is specified: *Reinforcement depth (A)* *Reinforcement depth (B)* *Reinforcement max force C (A)* *Reinforcement max force C (B)* *Reinforcement max force T (A)* *Reinforcement max force T (B)* *Reinforcement shear capacity*
Vehicle(s)	Modify only selected vehicles	N/A
Axle(s)	Modify only selected axles	N/A

Table 19.1: Property Editor functions

19.1.1 Fill element(s)

Limiting force

The limiting force is that force (kN per metre width) at which the fill (*i.e.* bar) element yields. This is a function of the specified soil properties (*i.e.* the product of the limiting fill pressure and the vertical block height).

To modify the limiting fill force, you must alter the **Fill force (H): max.** (see Section 19.1.2) value of the corresponding block entity.

19.1.2 Block(s)

Fill force (H): actual

This is the actual horizontal (passive) fill force applied to the block (kN per metre width). This will be zero:

1. before solving, and
2. in zones where the structure is moving away from the fill.

Fill force (H): max.

This is the maximum horizontal (passive) fill force that can be applied to the block (kN per metre width). It should be noted that the *actual* force applied may be lower.

For blocks that are associated with backing elements, the maximum fill force will be calculated assuming the default backing material properties.

Fill force (H): user-defined

This specifies whether the user is overriding the automatically calculated horizontal force applied to the block. If this is set to **true** then subsequent changes to the fill depth, unit weight etc. will not affect the fill horizontal force magnitude.

Fill force (V)

This is the fill vertical force applied to the block (kN per metre width). This force results from overlying fill, surface fill / ballast and track self weight loads

Fill force (V): user-defined

This specifies whether the user is overriding the automatically calculated vertical force applied to the block. Note that if set to true then subsequent changes to the fill depth, unit weight etc. will not affect the fill vertical force magnitude.

Support movement (rotation)

Rotational movement of support blocks can be modelled by entering a value (in radians) in this box. Rotations are measured in an anti-clockwise direction about the centroid of the block and become apparent once an analysis has taken place.

© *LimitState Ltd*

Support movement (x)

To model a horizontal (*x* direction) movement of a block, enter a value here. Settlement is measured in mm and shown following an analysis.

Support movement (y)

To model a vertical (*y* direction) settlement of a block, enter a value here. For a downwards movement, the value entered should be *negative*. Settlement is measured in mm and is shown following an analysis.

*Note: support movements cannot be specified without the selected block(s) first having restraint in the appropriate direction. For blocks without the required restraint, should a value be entered in a **Support movement...** box, LimitState:RING will give a warning that a restraint will also be added to the current selection (as well as the option to **cancel** this action).*

19.1.3 Contact(s)

Enabled

Specifies whether the selected contact(s) are active.

*Note: if this attribute is set to **false**, the contact will be assumed to be not present in the analysis and interpenetration can freely occur!*

Mortar loss

Mortar loss can affect the results of an analysis and should, therefore, be modelled as carefully as possible. In LimitState:RING, in an arch **Mortar loss (A)** refers to material removed from the contact on the intrados side of the arch, whilst **Mortar loss (B)** refers to material taken from the extrados side. Values for mortar loss are entered in mm.

Permit hinges

Specifies whether hinging failures are permitted at the selected contact(s).

Permit sliding

Specifies whether sliding failures are permitted at the selected contact(s).

© *LimitState Ltd*

Reinforcement properties

Reinforcement properties (see Chapter 17)

19.2 Using the explorers

In addition to the Property Editor, LimitState:RING has several explorers that allow the user to quickly check and compare the attributes of similar objects in the current project as shown in Figure 19.2.

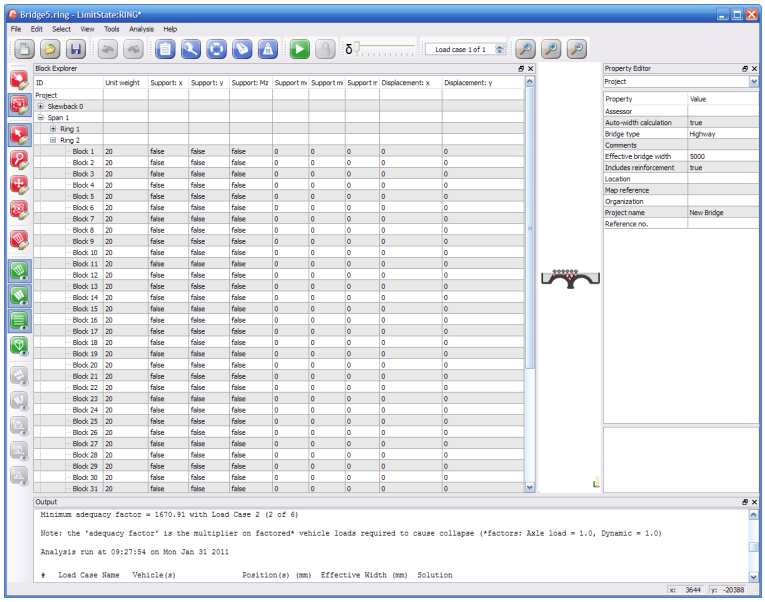


Figure 19.2: The LimitState:RING Block Explorer

19.2.1 Opening an explorer

To access the explorers, click on the **View** menu and highlight the **Explorers** option. There are four explorers to choose from:

1. Block explorer - displays the attributes of blocks contained in skewbacks, spans and piers.
2. Contact explorer - displays the attributes of contacts in spans and piers.
3. Vehicle explorer - displays the properties of each axle of every vehicle in the project.
4. Load case explorer - displays the properties of the vehicles according to their load case.

19.2.2 Navigating the explorers

Data in the explorers is presented in a convenient tabulated form, with objects grouped together in a logical manner. To begin using, simply expand the desired sections of the project *tree* on the left hand side of the explorer window (using the **+** and **-** buttons). Some of the columns may not be visible as the tables can be quite wide; for this reason, you may wish to widen the explorer window.

After using the mouse to select an object, group of objects or their properties in the explorer, the corresponding objects will be highlighted in the modeller window. This allows the user to determine precisely which parts of the bridge are being considered. Similarly, objects selected in the modeller window will be highlighted in the relevant explorer window.

The columns currently shown on a given explorer can be changed by right clicking with a mouse on the explorer title bar, selecting **View**, and then selecting and deselecting attributes as required.

19.2.3 Editing data

Editing data can be done in one of three ways:

1. By changing individual cells within the explorer.
2. By copying and pasting cells within the explorer.
3. By copying and pasting from a spreadsheet.

1. Changing individual cells

To change the contents of an individual cell in an explorer, double-click with the mouse. If the data is numerical, you will now be able to enter a new value using the keyboard. Alternately, if the data is an option (*e.g.*true / false), a drop-down list will appear containing all the available choices.

2. Copy and paste within an explorer

It is also possible to cut and paste data between several cells whilst within an explorer. To do this:

1. Use the mouse (or keyboard) to highlight the cells that you wish to copy.
2. Right-click to bring up the 20.5.3.
3. Select **Copy** (the copied cells will now have a dashed border).

© *LimitState Ltd*

4. Select the cells that you wish to paste into (the dimensions of the selected area should be the same as the copied area).
5. Right-click to bring up the explorer context menu.
6. Select **Paste** (the cells will now be filled with the new data).

An even quicker way of copying and pasting is to use the **Copy** / **Paste details** functions. These allow *entire* rows of data to be copied and pasted by selecting only the **ID** cell. To do this:

1. highlight the **ID** cell(s) of the object(s) to be copied.
2. Right-click to bring up the explorer context menu.
3. Select **Copy details** (the copied rows will now have a dashed border).
4. Select the **ID** cells that you wish to paste into (the dimensions of the selected area should be the same as the copied area).
5. Right-click to bring up the explorer context menu.
6. Select **Paste details** (the rows will now be filled with the new data).

Note:

1. *The standard keyboard shortcuts for **Copy** (CTRL+C) and **Paste** (CTRL+V) can also be used.*
2. *It is not possible to paste numerical data into a cell containing drop-down options (and* vice versa*)*
3. *It is not possible to select, copy or paste data to and from more than one feature at a time (e.g. in the Block explorer, you cannot copy data from an abutment and a span at the same time.)*
4. *It is not possible to overwrite object ID's using the **Copy** / **Paste details** functions (although the standard **Copy** / **Paste** will allow this).*

3. Copy and paste from a spreadsheet

To change the contents of many cells at the same time, it can be convenient to use a spreadsheet (such as Microsoft Excel). Firstly, making sure that you have your spreadsheet software open, highlight all the data that you wish to copy, then **Copy** (as described above). Navigate to your spreadsheet and **Paste** the data. You can then edit the data as required. To move the data back into LimitState:RING, the reverse process is carried out.

A convenient way to select all the data in the current branch (which is useful for exporting to a spreadsheet) is to use the **Copy all** function in a similar manner.

© *LimitState Ltd*

© LimitState Ltd

Chapter 20

Display options

20.1 General

20.1.1 Language specific variations

It should be noted that the different language options in LimitState:RING can cause the display to alter according to the prevailing direction of reading. Here, it is assumed that the English Language version is being used; sections of the LimitState:RING window referred to as right and left should be reversed where appropriate.

20.1.2 Scrollbars

Vertical and horizontal scrollbars allow the display area to be shifted in the vertical and horizontal sense respectively.

20.1.3 Current mouse position

The coordinates of the mouse are shown in the bottom right hand corner of the screen. This may be useful for determining the global position of various parts of a bridge (the datum for all bridges is the left hand springing of span 1).

20.1.4 Scrolling wheels

Most mice are equipped with a third button that is used for scrolling. LimitState:RING makes use of this additional feature by allowing the user to pan and zoom the display:

- To **pan**, simply press and hold the third button whilst in the display window. Moving the mouse will now pan the image around the screen.

- To **zoom**, simply roll the wheel up to zoom out and down to zoom in.

The pan and zoom functions are also accessible via the Table 20.8.

20.2 Viewer pane

In order to better visualize the model and post-analysis failure mechanism, it is possible to manipulate the view in a number of ways:

20.2.1 Rotating the model

The rotate tool

The rotate tool allows the user to quickly and easily manipulate the scene in the viewer pane by providing handles to rotate around the major cartesian axes and also freely in 3D.

To access the tool, click the **Rotate** icon on the **Cursor** toolbar, , or open the **Viewer pane** context menu by right-clicking the mouse anywhere within the viewer pane and selecting **Rotate** $>$ **Rotate**. This action will overlay the rotate tool on top of the viewer pane (Figure 20.1).

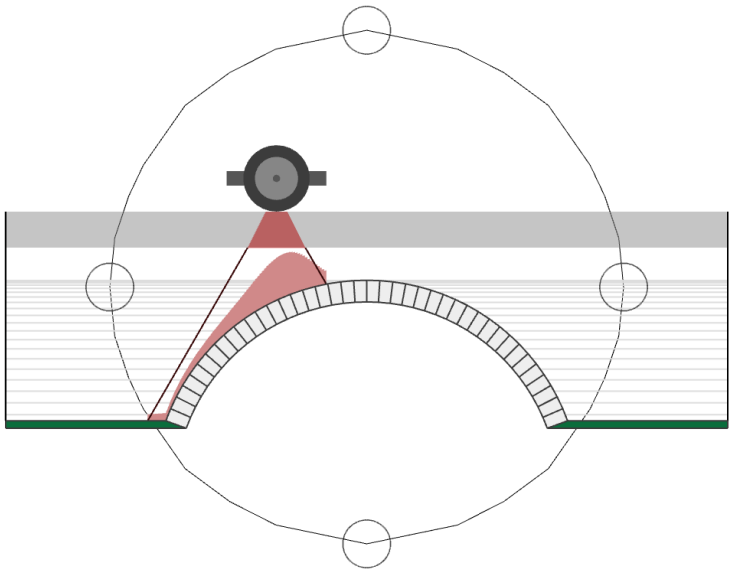


Figure 20.1: The rotate tool

Now, when hovering in the different areas of the viewer pane, rotate cursors are displayed:

Rotate x Hovering the cursor in the small circles at the top or bottom brings up the **rotate x** cursor, . Click with the left mouse button and hold. Moving the mouse up and down the screen will now rotate the model around the x-axis.

Rotate y Hovering the cursor in the small circles at the left or right brings up the **rotate y** cursor, . Click with the left mouse button and hold. Moving the mouse left and right across the screen will now rotate the model around the y-axis.

Rotate z Hovering the cursor outside the large central circle brings up the **rotate z** cursor, . Click with the left mouse button and hold. Moving the mouse up and down the screen will now rotate the model around the z-axis.

Rotate 3D Hovering the cursor inside the large central circle brings up the **rotate 3D** cursor, . Click with the left mouse button and hold. Moving the mouse in any direction (keeping within the circle) will now rotate the model freely in any direction.

To exit the **Rotate tool** select an alternative cursor option in the **Cursor** toolbar.

Rotate cursors

In addition to the **Rotate tool**, the model can be rotated by accessing any of the rotate cursors (**rotate x**, **rotate y** etc.) individually. To do this, right-click in the **Viewer pane** to bring up the context menu, then select **Rotate** and choose from one of the four options beneath the horizontal line (as described above). The cursor will then change to match the chosen option. Clicking and holding the left mouse button then moving the cursor anywhere within the viewer pane will rotate the model in the preferred manner. To exit the **Rotate tool** select an alternative cursor option in the **Cursor** toolbar.

Predefined viewpoints

For quick inspection, it is possible to view the model in the viewer pane from one of a number of predefined 3D viewpoints. To change the current view to one of these, right-click the mouse in the **Viewer pane** to bring up the context menu, then select **View** and choose from one of the following options:

Top View the model from above - in the negative y direction.

Bottom View the model from below - in the positive y direction.

Right View the model from the right - in the negative x direction.

Left View the model from below - in the positive x direction.

Front View the model from the front - in the positive z direction. This is the default view.

Back View the model from the back - in the negative z direction.

Alternately, these functions can be accessed by navigating to the **View** menu and selecting one of the options under **3D View**.

© *LimitState Ltd*

3D and perspective views

The scene can be quickly set to a default 3D viewpoint by right-clicking in the **Viewer pane** to bring up the context-menu then selecting **View** > **3D View**. The model will then appear viewed from above and to the right (at 45°) looking toward the origin.

To view the model in perspective, select the **Toggle perspective** icon, , from the **View** toolbar. Alternatively, this function can be accessed from the **Viewer pane** context menu (**View** > **Toggle Perspective**).

Both the **3D View** and **Toggle Perspective** functions can be accessed from the **View** > **3D View** menu.

20.3 Menus

20.3.1 File menu

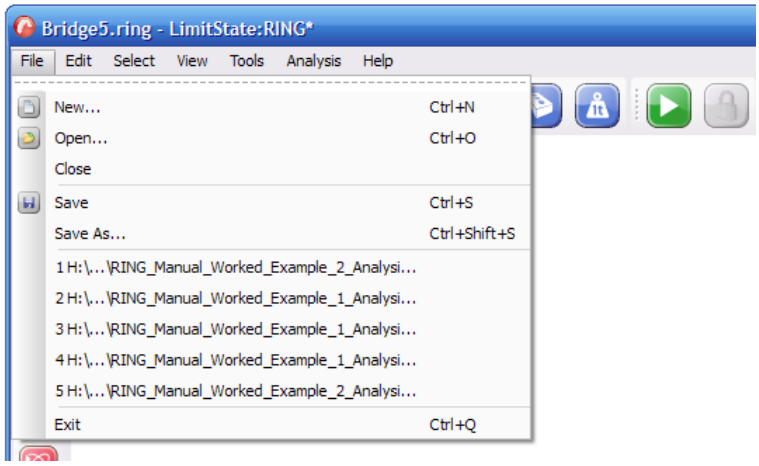


Figure 20.2: The LimitState:RING**File menu**

Function	Shortcut	Description
New...	Ctrl+N	Create a new bridge project
Open...	Ctrl+O	Open an existing bridge project
Close		Close the current bridge project
Save	Ctrl+S	Save the current bridge project
Save As...	Ctrl+Shift+S	Save the current bridge project under a specified name
Open recent file		Open one of the 5 most recently accessed files
Exit	Ctrl+Q	Exit LimitState:RING

Table 20.1: **File menu** functions

20.3.2 Edit menu

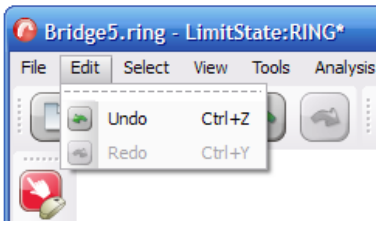


Figure 20.3: The LimitState:RING **Edit menu**

Function	Shortcut	Description
Undo	Ctrl+Z	Step back to the point immediately before the last action was taken
Redo	Ctrl+Y	Redo a previously undone action

Table 20.2: **Edit menu** functions

20.3.3 Select menu

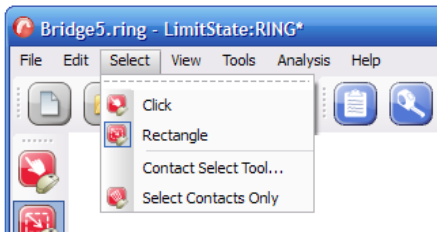


Figure 20.4: The LimitState:RING **Select menu**

Function	Description
Click	Select a single object (or multiple objects using CTRL)
Rectangle	Select multiple objects within a rectangular zone
Contact Select Tool	Opens a dialog allowing you to select multiple contacts.
Select Contacts Only	Select only contact elements

Table 20.3: **Select menu** functions

20.3.4 View menu

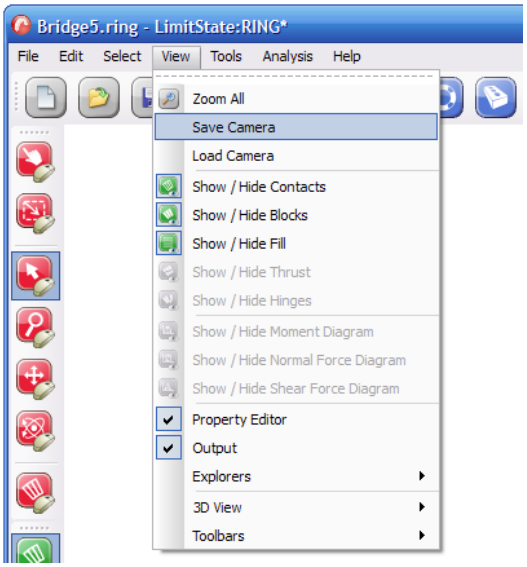


Figure 20.5: The LimitState:RING **View menu**

Function	**Description**
Zoom All	View the extents of the bridge and loading
Save Camera	Remember the current camera viewpoint for later use
Load Camera	Load the previously saved camera viewpoint
Show / Hide Contacts	Toggle the display of the contact surfaces
Show / Hide Blocks	Toggle the display of the blocks
Show / Hide Fill	Toggle the display of the fill (bar) elements
Show / Hide Thrust*	Toggle the display of the thrust zone
Show / Hide Hinges*	Toggle the display of the hinges
Show / Hide Moment Diagram *	Toggle the display of the moment diagram
Show / Hide Normal Force Diagram*	Toggle the display of the normal force diagram
Show / Hide Shear Force Diagram*	Toggle the display of the shear force diagram
Property Editor	Toggle the display of the 'Property Editor' pane (default position: right of screen)
Output	Toggle the display of the 'Output' pane (default position: bottom of screen)
Explorers	Toggle the display of the Block / Contact / Vehicle and **Load case explorers** (see Section 19.2)
3D View	Choose from a variety of pre-defined camera viewpoints
Toolbars	Toggle the display of default and non-default toolbars

Table 20.4: **View menu** functions (* = post-solve only)

20.3.5 Tools menu

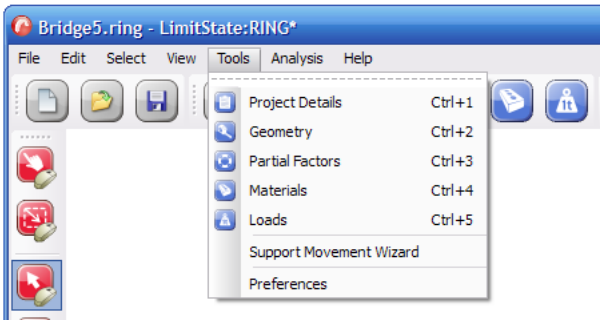


Figure 20.6: The LimitState:RING **Tools menu**

Function	Description
Project Details	Open the **Project details** dialog box
Geometry	Open the **Geometry** dialog box
Materials	Open the **Materials** dialog box
Partial Factors	Open the **Partial factors** dialog box
Loads	Open the **Loading** dialog box
Support Movement Wizard	Open the **Support Movement Wizard**
Preferences	Modify General / Report preferences

Table 20.5: **Tools menu** functions

20.3.6 Analysis menu

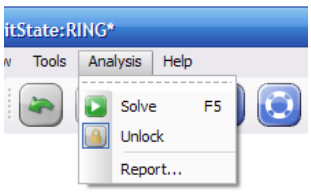


Figure 20.7: The LimitState:RING **Analysis menu**

Function	Shortcut	Description
Solve	F5	Analyse the current problem
Unlock		Unlock the project for editing
Report...		View the analysis report

Table 20.6: **Analysis menu** functions

20.3.7 Help menu

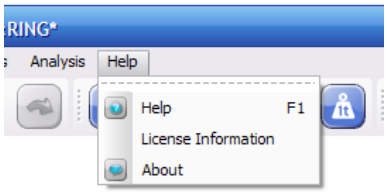


Figure 20.8: The LimitState:RING **Help menu**

Function	Shortcut	Description
Help	F1	Enter the Help system
License Information		Open the license information dialog
About		Display LimitState:RING version details

Table 20.7: **Help menu** functions

20.4 Toolbars

20.4.1 Default toolbars

By default, the toolbars listed in Table 20.8 are displayed when you open LimitState:RING:

Toolbar	Functions
Analysis	Solve / Unlock
Cursor	Click / Rectangle / Select / Zoom / Pan
Edit	Undo / Redo / Delete
File	New / Open / Save
Load Cases	Load case spin box
Magnification	Displacement magnification slider
View	Toggle Perspective Show Contacts / Show Bar Elements / Show Thrust Line Show Blocks
Zoom	Zoom All / Zoom In / Zoom Out
Properties	Project Details / Geometry / Partial factors Materials / Loads

Table 20.8: Default toolbars

20.4.2 Optional toolbars

To access some of the less commonly utilized features of LimitState:RING, it may be necessary to open a separate toolbar. To do this, click **View** and select **Toolbars**. You will now have the option to open one of the toolbars listed in Table 20.9.

Toolbar	Functions
Cursor 3D	Rotate the model in 3D: Rotate about x / Rotate about y / Rotate about z
View 3D	View the model in 3D: Top / Bottom / Left / Right / Front / Back
Help	Help / About

Table 20.9: Optional toolbars

20.5 Context menus

Depending upon the position of the cursor, right clicking the mouse within the LimitState:RING environment will bring up one of several context menus:

20.5.1 Viewer pane context menu

Right-clicking within the viewer pane will bring up the menu shown in Figure 20.9.

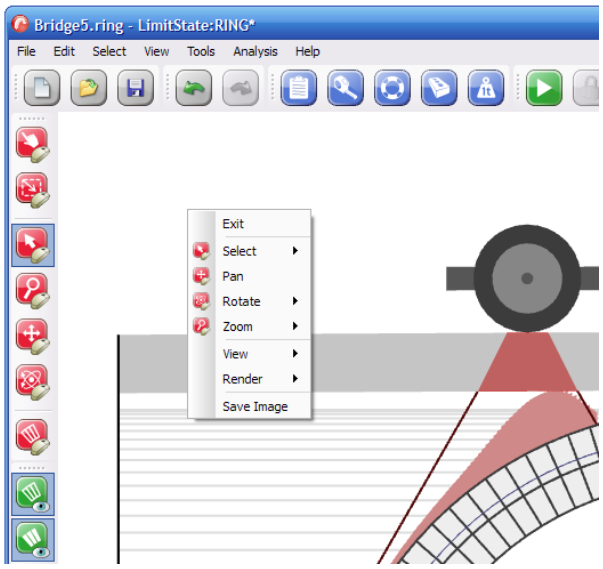


Figure 20.9: Viewer pane context menu

From here, you can easily access many of the display-related functions of the toolbars, as well as several other independent functions listed in Table 20.10.

Toolbar	Functions
Exit	Exits the context menu
Select	Access **Select menu** (see Section 20.3.3) functions
Pan	Pan around the viewing window
Rotate	Access the **Cursor 3D** (see Section 20.4.1) toolbar functions
Zoom	Access the **View** (see Section 20.4.1) toolbar functions
View	Access the **View 3D** (see Section 20.4.2) toolbar functions
Render	Change the quality of rendering in the viewer window
Save image	Save the current view as an image (*png*, *jpg*, *ps* or *tiff*)

Table 20.10: Viewer pane context menu options

20.5.2 Toolbar / property editor context menu

Right-clicking within any toolbar or at the top of the **Property Editor** (see Section 19.1) will bring up the following menu depicted in Figure 20.10.

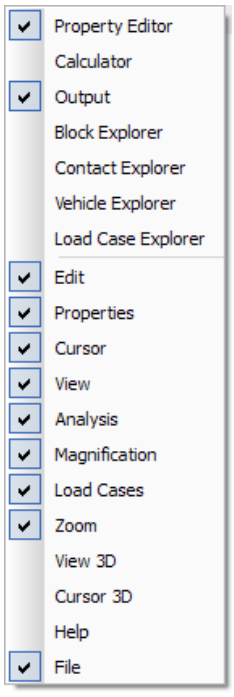


Figure 20.10: Toolbar / Property Editor context menu

From here, it is possible to toggle the displaying of the **Property editor** (see Section 19.1), **Output Pane** (see Section 11.7), **Explorers** (see Section 19.2), and **Toolbars** (see Section 20.4).

20.5.3 Explorer context menu

Right-clicking within any explorer will bring a menu of the type shown in Figure 20.11.

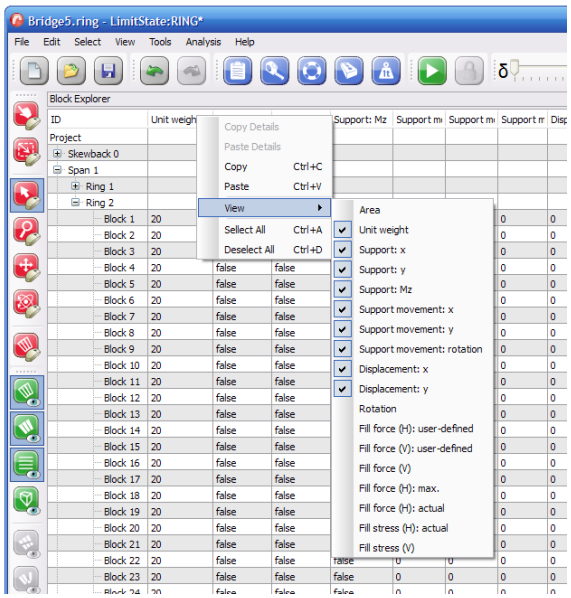


Figure 20.11: Explorer context menu

From here you can copy and paste sections of data or, by selecting **View**, determine which properties are displayed as columns in the explorer.

© LimitState Ltd

Chapter 21

Analysis

To perform an analysis, on the **Analysis** menu click **Solve**. Alternatively this command can be accessed via the solve button on the toolbar , and the keyboard shortcut for the command is **F5**.

21.1 Auto-solve

If the user finds that they use the **Drag and solve** (see Section 4.1.7) method for the majority of their problems, the **Auto-solve** feature should be enabled. This will cause LimitState:RING to automatically solve each time the vehicle is moved. To enable, check the **Solve automatically...** box in the **Preferences** dialog (located in the **Tools** menu)

21.2 Types of analysis

21.2.1 Normal analysis

When a single load case is specified, the load factor that would, when applied to the specified live loading, cause the bridge to collapse is calculated and displayed. When multiple load cases are set up the collapse load factor associated with each case is calculated in turn. However, as the majority of the problem remains unchanged, the total CPU time required for two load cases (say) is rather less than twice that required for one load case.

21.2.2 Iterative analysis

When finite masonry crushing strength is specified, the governing contact moment vs. normal force failure envelope is non-linear, which means that an iterative analysis is required. In the iterative analysis the failure envelope is progressively refined until the true non-linear failure envelope is properly represented (by using a series of linear constraints).

By default, the intermediate output from an iterative analysis is suppressed and only the final adequacy factor is shown. To override this, and display *all* the iteration data in the output pane, check the **Display iteration information in output window** box in the **Preferences** dialog as shown in Figure 21.1.

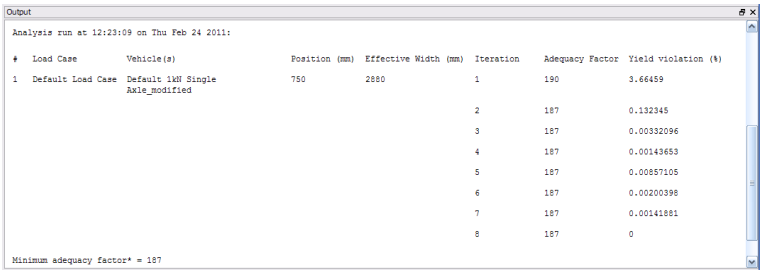


Figure 21.1: Analysis details in the output pane

Note: when multiple load cases are also specified an iterative analysis is now performed for all load cases.

By default, the displayed output from an analysis is restricted to:

- The load case **number** (#)
- The **load case name**
- The names of the **vehicle(s)** in the load case
- The vehicle **position(s)** (in mm, measured from the left-hand springing of the left-hand arch)
- The **effective bridge width** (in mm, either fixed or calculated)
- The **solution** (as an adequacy factor on the applied load)
- The **minimum adequacy factor** calculated over all the load cases

To enable the display of *all* iteration data, go to the **Preferences** dialog in **Tools** (see Section 20.3.5) and select the option to ***Display iteration information in output window***.

21.3 The solvers

A solver is required to find the critical collapse load factor and associated collapse mechanism. The internal forces in the structure must satisfy all specified yield constraints; these are set up for a particular problem by LimitState:RING.

Version 3.2.a of LimitState:RING makes use of two third-party solvers - Mosek (which uses an interior point optimization algorithm) and CLP (which uses a simplex optimization algorithm). Both are powerful linear programming solvers which are called as a subroutine and, to maximize efficiency, the problem data is passed *via* memory.

By default, LimitState:RING will automatically choose the most appropriate solver for the type of problem being analysed. A single ring problem (one where all the spans possess a single ring of masonry) will be solved using CLP. Multi-ring problems will be solved using Mosek. Should the user wish to override this setting, it can be done in the **Preferences** dialog (**Tools** >**Preferences**). Note that solving complex multi-ring problems using CLP may result in a long computation time.

21.4 Analysis results

21.4.1 Adequacy factor found

Following a successful analysis the minimum **Adequacy factor** will be displayed at the bottom right of the LimitState:RING application window, and also in the **Output Pane**. If multiple load cases are specified then this load factor will be the lowest found for all the load cases tried.

The **Adequacy factor** is the multiplier on factored vehicle loads required to cause collapse in the structure being modelled.

21.4.2 No solution found

When crushing of the masonry is not enabled it is possible that the applied load can be increased without limit. In this case the structure can be described as being 'geometrically locked'. This result will typically occur if the specified arch thickness is large and rigid abutments are specified. Alternatively if no part of an applied load falls on the bridge then this outcome will result.

Alternatively it might be that no solution could be found because no viable equilibrium state could be identified. This result will typically occur if the arch is the 'wrong shape' in relation to the specified dead loading (i.e. the dead loads alone are sufficient to cause the structure to collapse). In this case an 'unstable' message will be reported.

21.4.3 Aborting an analysis

After an analysis is started, LimitState:RING then waits for a solution to be found. To abort this process, the user should click on the red stop button on the toolbar or press the **Esc** key to abort the analysis and return control to the user.

© *LimitState Ltd*

© LimitState Ltd

Chapter 22

Post analysis functions

22.1 Visual output

Once a problem has successfully solved, visual output is displayed on the viewing pane. The thrust line shown is that calculated without accounting for the presence of reinforcement, i.e:

Eccentricity = Axial Force / Moment

Depth of crushing = Axial Force / (Masonry crushing strength x Bridge Width)

It follows that, when the reinforcement is active, the thrust-line will lie outside the barrel thickness, as indicated in Figure 22.1.

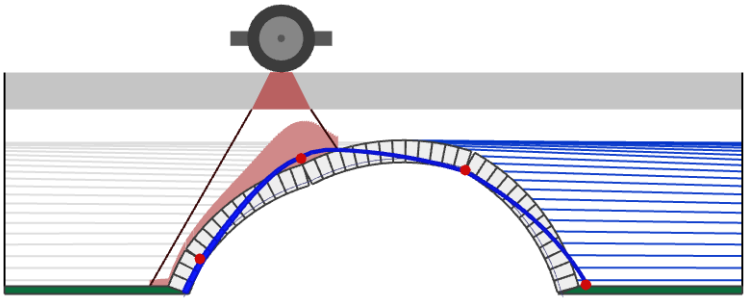


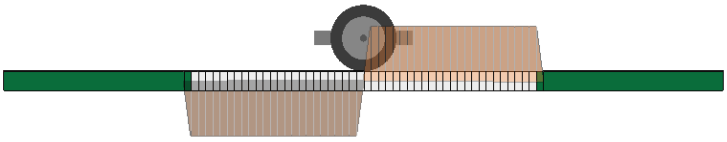
Figure 22.1: Thrust line lying outside the arch barrel in areas where reinforcement is active

Note: in the vicinity of backing the thrust-line may exhibit spikes as large forces are transmitted from the arch into the backing material. Also, if reinforcement is specified in the problem, the line of thrust may be displayed as being outside of the arch but without an associated hinge forming. Both of these situations are perfectly normal.

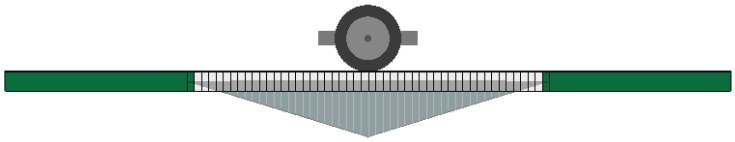
22.1.1 Force diagrams

To allow alternative interpretation of the behaviour of the structure when reinforcement is present, in LimitState:RING the capability to view normal force (kN/m bridge width), shear force (kN/m bridge width) and bending moment (kNmm/m bridge width) diagrams has been added. These can be accessed in a solved model by clicking the moment, shear or normal force icons: , and .

This is illustrated for the case of a beam in Figure 22.2, and in the case of an arch bridge in Figure 22.3.



(a) Shear force



(b) Moment

Figure 22.2: Sample beam shear force and moment diagrams

The force diagrams can be displayed by navigating to the relevant options in the **View** menu, or by using the appropriate toolbar buttons to toggle the view of the ***normal force***, ***shear force*** and ***moment***.

Note:

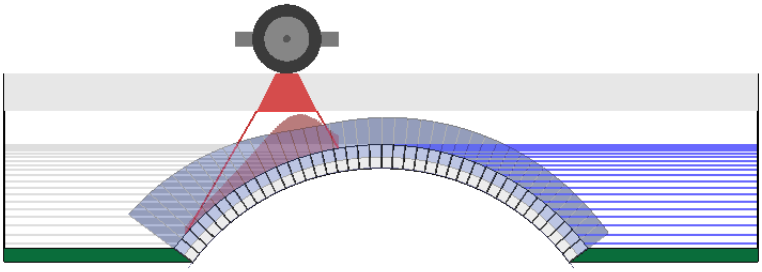
1. *The entities (e.g. thrust and/or bending moment diagrams) displayed on screen when the report is generated (via Analysis > Report...) will be those also displayed on the image in the report.*
2. *A given magnitude (e.g. shear force magnitude) can be queried by clicking on the part of the contact of interest not obscured by the diagram, and referring to the Property Editor.*

22.2 Quantitative output

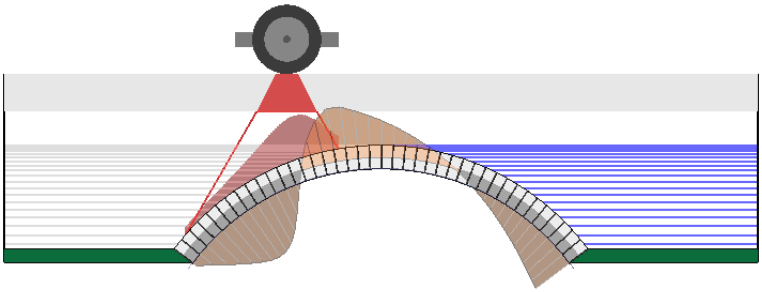
The contact ***normal force*** and ***bending moment*** values displayed in the **Property Editor**, **Contact Explorer** and in the **Report Output** are combined values that take account of joint stresses and the presence of reinforcement.

Full details of the reinforcement specifications are also included in the **Report Output** (see Chapter 23).

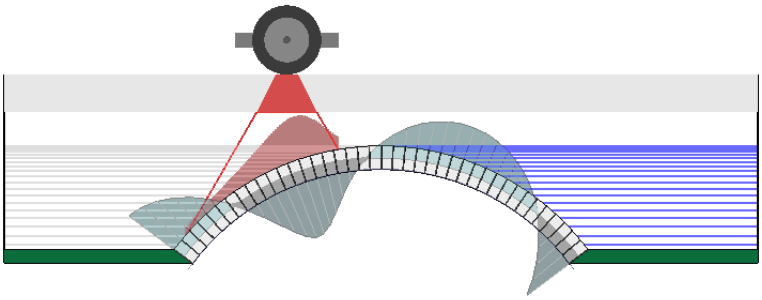
© *LimitState Ltd*



(a) Normal forces



(b) Shear forces



(c) Moments

Figure 22.3: Sample arch bridge normal force, shear force and moment diagrams

© *LimitState Ltd*

© LimitState Ltd

Chapter 23

Report output

23.1 Viewing report output

Following an analysis it is often useful to summarize details of the bridge and of the analyses performed, and to then print this out. LimitState:RING comes with an inbuilt word processor, so report output can be generated on any computer, exported to an Adobe pdf file and / or printed.

To view the report as shown in Figure 23.1, on the **Analysis** menu click **Report...**.

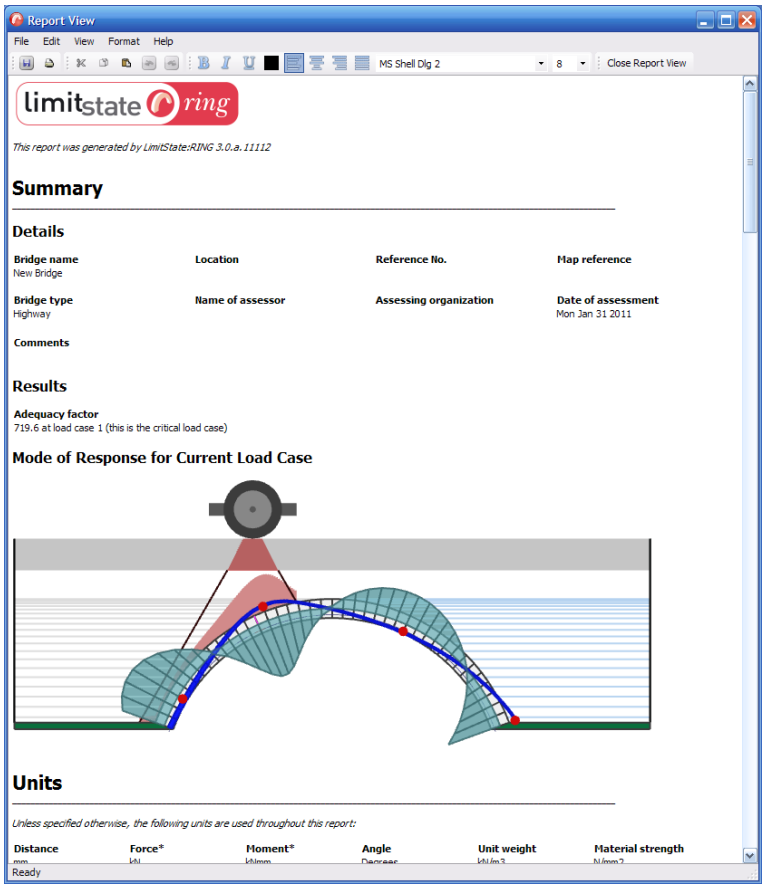


Figure 23.1: LimitState:RING report

From here, details can be changed as required, or the layout altered, using commands familiar to most users who have previously used word-processing packages.

23.2 Adding a template, header or footer

A style template or custom headers and footers can also be appended to the report by selecting the **Tools** menu, clicking **Preferences** and selecting the **Report** tab as shown in Figure 23.2.

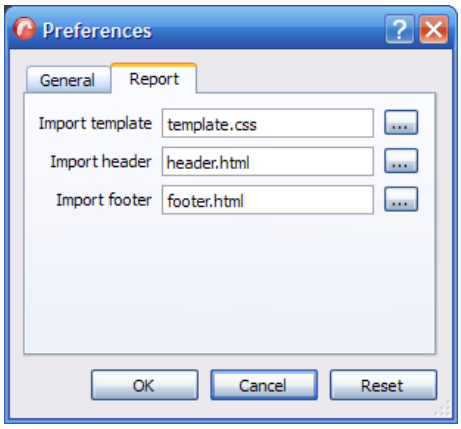


Figure 23.2: LimitState:RING report preferences

© LimitState Ltd

Part V

Appendices

Appendix A

Mathematical formulation

A.1 Joint equilibrium formulation (adequacy factor analysis)

This section contains details of the mathematical formulation used in LimitState:RING. A joint equilibrium formulation, similar to that proposed initially by Livesley (1978) is used. Whilst this formulation produces a large number of constraints and variables, the total number of non-zero elements will generally be relatively small, which means that it can be solved very efficiently using modern interior point Linear Programming (LP) algorithms. Thus assuming there are b blocks and c contact surfaces, the problem may be stated as follows

$$\text{Max } \lambda \tag{A.1}$$

subject to the equilibrium constraints:

$$\mathbf{Bq} - \lambda \mathbf{f}_L = \mathbf{f}_D \tag{A.2}$$

and no-tension ('rocking') yield constraints:

$$\left.\begin{array}{c} m_i \leq 0.5 n_i t_i \\ m_i \geq -0.5 n_i t_i \end{array}\right\} \text{for each contact, } i = 1, ..., c \tag{A.3}$$

and sliding yield constraints:

$$\left.\begin{array}{c} s_i \leq \mu_i t_i \\ s_i \geq -\mu_i t_i \end{array}\right\} \text{for each contact, } i = 1, ..., c \tag{A.4}$$

where λ is the load factor (which is the same as the adequacy factor AF when the vehicle loads are not pre-factored), $\mathbf{B}$ is a suitable $(3b \times 3c)$ equilibrium matrix containing the direction cosines and $\mathbf{q}$ and $\mathbf{f}$ are respectively vectors of contact forces and block loads. Thus $\mathbf{q}^T = \{n_1, s_1, m_1, n_2, s_2, m_2, ..., n_c, s_c, m_c\}; \mathbf{f} = \mathbf{f}_D + \lambda \mathbf{f}_L$ where $\mathbf{f}_D$ and $\mathbf{f}_L$ are respectively vectors of dead and live loads. Contact and block forces, dimensions and frictional properties are shown

in Figure A.1. Using this formulation the LP problem variables are the contact forces: n_1, s_1, m_1 (where $n_i \geq 0; s_i, m_i$ are unrestricted 'free' variables), and the unknown load factor, λ.

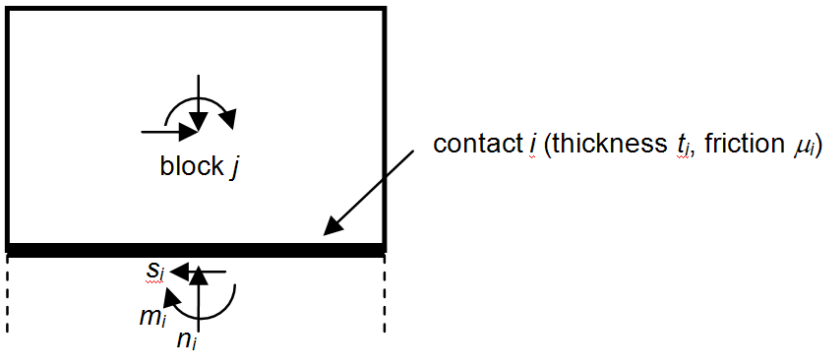


Figure A.1: Block and contact forces

A.2 Joint equilibrium formulation (support movement analysis)

The problem may be stated as follows

$$\text{Min } E \tag{A.5}$$

subject to the equilibrium constraints:

$$\mathbf{Bq} - \mathbf{f}_{SUP} = \mathbf{f}_D \tag{A.6}$$

and a constraint specifying support movement:

$$\mathbf{d}_{SUP}^T \mathbf{f}_{SUP} - E = 0 \tag{A.7}$$

where E is the support movement energy, $\mathbf{d}_{sup}$ and $\mathbf{f}_{sup}$ are respectively vectors of pre-defined block support movements and unknown block support reactions (see Figure A.1 for block degrees of freedom). Other terms are as defined in Appendix A.1.

The yield conditions equation A.3 and equation A.4 are also be imposed in unchanged form. Using this formulation the LP problem variables are the contact forces: (where $n_i \geq 0; s_i, m_i$ are unrestricted 'free' variables), the support reactions in $\mathbf{f}_{sup}$ and the support movement energy E.

A.3 Including finite masonry material strength

The yield constraints (equation A.3) given in Appendix A.1 are valid only if the material is infinitely strong in compression. If it is assumed that the masonry possesses finite masonry

© LimitState Ltd

strength and that the thrust is transmitted through a rectangular crush block, then (equation A.3) may be replaced with:

$$\left.\begin{aligned} m_i &\leq n_i\left(0.5t_i - \tfrac{n_i}{2\sigma_{crush}b}\right) \\ m_i &\geq -n_i\left(0.5t_i - \tfrac{n_i}{2\sigma_{crush}b}\right) \end{aligned}\right\} \text{for each contact, } i = 1, ..., c \tag{A.8}$$

To aid comparison, both equation A.3 and equation A.8 are plotted in Figure A.2:

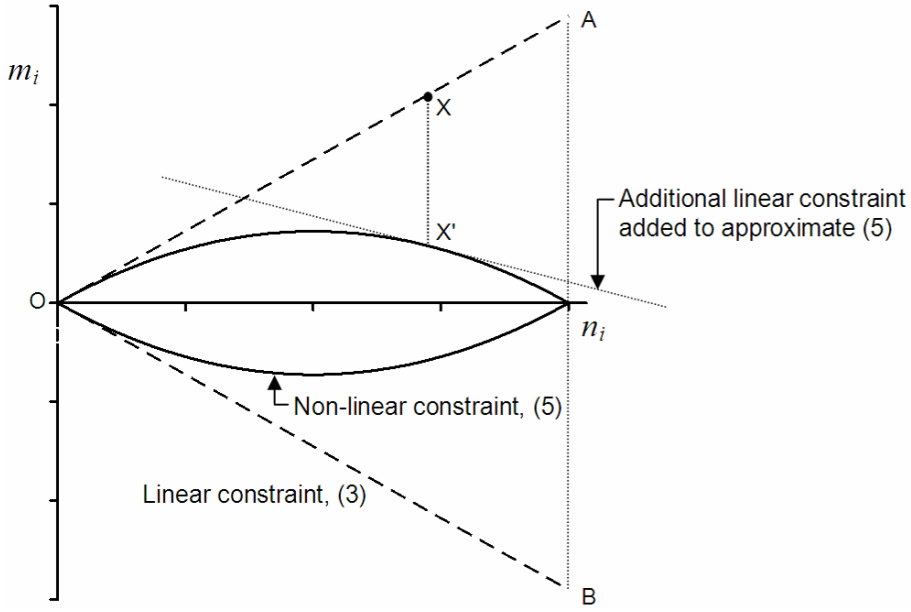


Figure A.2: Contact surface moment vs. normal force failure envelopes

However, the constraints in equation A.8 are non-linear. Therefore if a Linear Programming (LP) solver is still to be used to obtain a solution to the global problem, then these constraints need to be approximated as a series of linear constraints.

In order to minimise the number of constraints in the problem (and to maximise computational efficiency) an iterative solution algorithm which involves only refining the representation of the failure envelope where required is used:

A.3.1 Algorithm

1. For each contact i, initially add three linear constraints (i.e. OA, OB & AB in Figure A.2).
2. Obtain a solution to the global LP problem
3. For each contact i, substitute n_i from the last solution into the inequality constraints in equation A.8. If a constraint is violated, calculate the violation factor k_i. i.e.

$$k_i = \frac{|m_i|}{n_i\left(0.5t_i - \frac{n_i}{2\sigma_{crush}b}\right)}$$

© LimitState Ltd

4. For each contact with $k_i > 1.0$ (i.e. violation), add an additional linear constraint (e.g. in the case of point X in Figure A.2, introduce a new linear constraint tangential to the true non-linear constraint at X').

5. Repeat from step 2 until the maximum value of $k_i < 1 + tol$, where tol is taken as a suitably small value.

A.4 Including reinforcement

LimitState:RING uses rigorous optimization techniques to find the distribution of internal forces which give rise to the largest possible load factor which can be applied whilst still satisfying equilibrium and yield constraints.

When reinforcement is involved, consider a contact between adjacent rigid blocks. In order to model a rectangular crush block of initially unknown depth (see Figure A.3), the failure criterion given in Appendix A.3 can still be used providing the normal force, n_{cont}, and moment, m_{cont}, values used account only for the effects of contact pressures. i.e. which exclude the effects of any reinforcement bars which span across the contact.

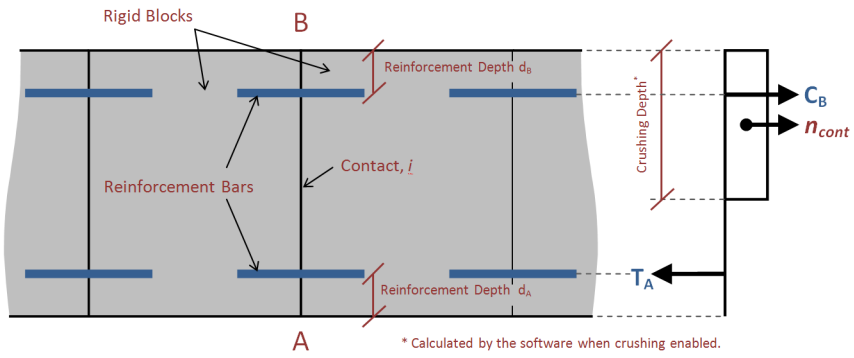


Figure A.3: Contact surfaces with top and bottom reinforcement bars

However the presence of reinforcement has the effect of modifying the total normal force and moment at the contact as follows:

Total normal force carried, $n_{tot} = n_{cont} + C_A + C_B - T_A - T_B$

Total moment carried, $m_{tot} = m_{cont} + d_A(C_A - T_A) + (t - d_B)(C_B - T_B)$

Where C_A, T_A and C_B, T_B are respectively the compressive and tensile reinforcement forces near contact edges A and B, and t is the contact thickness. The reinforcement forces are free to take on any positive value up to a user-specified limit the software will find the values corresponding to the critical load factor. The increased normal force and moment which can be carried means that the predicted load carrying capacity of a bridge containing reinforcement will always be greater than or equal to that corresponding to an otherwise identical bridge without reinforcement.

Modelling bars separately from the line contact as described above obviates the need to use a

more complex 'reinforced concrete' type yield surface. However, the equivalence of the results obtained can easily be verified. (See Appendix F.)

Finally, it should be noted that the contact normal forces and moments reported in LimitState:RING (in the **Property Editor**, **Explorers** and **Report**) are total values, i.e. as given by n_{tot} and m_{tot} respectively.

© *LimitState Ltd*

A.5 Worked example

This worked example is designed to help illustrate how the mathematical formulation described in Appendix A.1 may be applied in practice.

To ensure that the number of problem variables and constraints are kept down to an absolute minimum a simple 3 block arch is considered, as shown in Figure A.4. (This geometry can be generated in LimitState:RING by specifying a span of 20m, a rise of 10m, a ring thickness of 1.5m and 3 blocks in the span.) Each block, labelled A, B and C, is for simplicity assumed to weigh 1 unit. Block A is also subject to a unit live load applied at the centroid. The blocks are separated by four contact surfaces, labelled 1, 2, 3 and 4. The blocks are taken to be infinitely strong and the contacts are taken to have a coefficient of friction of 0.6. The objective is to find the load factor (or 'multiplier') λ which can be applied to the live load in order to cause collapse. (N.B. The load factor λ is equivalent to the Adequacy Factor AF when the live load is not pre-factored).

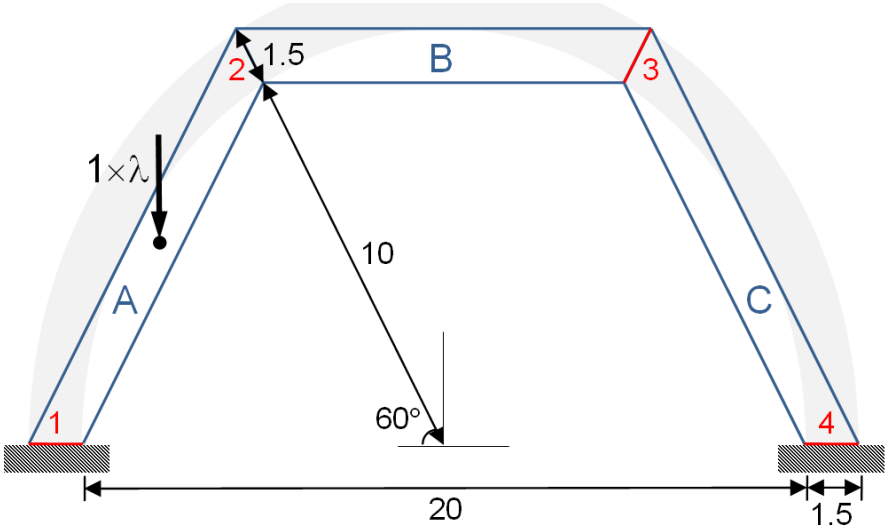


Figure A.4: Details of simple 3 block arch example problem

As described in Appendix A.1, the problem formulation involves equilibrium constraints, yield constraints and an objective function.

A.5.1 Equilibrium constraints

Consider the equilibrium of block A a shown in Figure A.5.

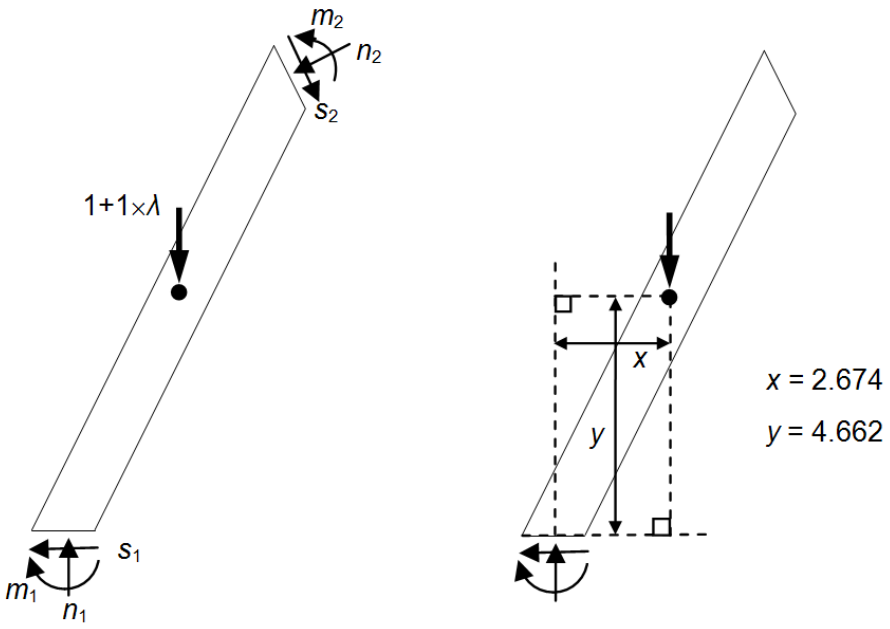


Figure A.5: Block A: forces and geometry for formulating equilibrium constraints

Resolving forces in the X direction:

$$-s_1 + s_2 \sin 30 - n_2 \cos 30 = 0 \tag{A.9}$$

$$\therefore -s_1 + \frac{s_2}{2} - n_2 \frac{\sqrt{3}}{2} = 0 \tag{A.10}$$

Resolving forces in the Y direction:

$$-n_1 + s_2 \cos 30 + n_2 \sin 30 + \lambda + 1 = 0 \tag{A.11}$$

$$\therefore -n_1 + s_2 \frac{\sqrt{3}}{2} + \frac{n_2}{2} + \lambda + 1 = 0 \tag{A.12}$$

Taking moments about the block centroid:

$$-s_1 \times y - n_1 \times x - m_1 - s_2 \times y + n_2 \times x + m_2 = 0 \tag{A.13}$$

$$\therefore -4.662 s_1 - 2.674 n_1 - m_1 - 4.662 s_2 + 2.674 n_2 + m_2 = 0 \tag{A.14}$$

Clearly similar relationships can readily be derived for blocks B and C (in the case of the moment equilibrium constraint it is only necessary to change the subscripted numerals).

© LimitState Ltd

A.5.2 Yield constraints

Consider the no-tension constraints for contact 1. These can be obtained simply by entering the actual arch thickness (1.5) in equation A.3, and re-arranging to ensure problem variables are on the left hand side:

$$m_1 \leq 1.5(0.5n_1) \tag{A.15}$$

$$\therefore -1.5(0.5n_1) + m_1 \leq 0 \tag{A.16}$$

$$m_1 \geq 1.5(-0.5n_1) \tag{A.17}$$

$$\therefore -1.5(-0.5n_1) - m_1 \leq 0 \tag{A.18}$$

Consider the sliding constraints for contact 1: These can be obtained simply by entering the actual coefficient of friction (0.6) in equation A.4 and re-arranging to ensure problem variables are on the left hand side:

$$s_1 \leq 0.6n_1 \tag{A.19}$$

$$\therefore s_1 - 0.6n_1 \leq 0 \tag{A.20}$$

$$s_1 \geq -0.6n_1 \tag{A.21}$$

$$\therefore -s_1 - 0.6n_1 \leq 0 \tag{A.22}$$

These yield constraints can also be used for contacts 2, 3 and 4 (by simply changing the subscript according to the contact number).

A.5.3 Objective function

In this case the objective is simply to maximise the load factor, λ.

A.5.4 Problem matrix

The problem matrix (also termed the 'linear programming tableau') may now be formed, as shown in Table A.1.

© *LimitState Ltd*

			Main Problem variables (forces)														
			Contact 1			Contact 2			Contact 3			Contact 4					
			Shear s_1	Normal n_1	Moment m_1	Shear s_2	Normal n_2	Moment m_2	Shear s_3	Normal n_3	Moment m_3	Shear s_4	Normal n_4	Moment m_4	λ		
Equilibrium constraints	Block A	X	-1			0.5	-0.866									=	0
		Y		-1		0.866	0.5								1	=	-1
		Moment	-4.662	-2.674	-1	-4.662	2.674	1								=	0
	Block B	X				-0.5	0.866		-0.5	-0.866						=	0
		Y				-0.866	-0.5		0.866	-0.5						=	-1
		Moment				-4.662	-2.674	-1	-4.662	2.674	1					=	0
	Block C	X							0.5	0.866		-1				=	0
		Y							-0.866	0.5			-1			=	-1
		Moment							-4.662	-2.674	-1	-4.662	2.674	1		=	0
Yield constraints	Contact 1	No-tension+		-0.75	1											≤	0
		No-tension-		-0.75	-1											≤	0
		Sliding+	1	-0.6												≤	0
		Sliding-	-1	-0.6												≤	0
	Contact 2	No-tension+					-0.75	1								≤	0
		No-tension-					-0.75	-1								≤	0
		Sliding+				1	-0.6									≤	0
		Sliding-				-1	-0.6									≤	0
	Contact 3	No-tension+								-0.75	1					≤	0
		No-tension-								-0.75	-1					≤	0
		Sliding+							1	-0.6						≤	0
		Sliding-							-1	-0.6						≤	0
	Contact 4	No-tension+											-0.75	1		≤	0
		No-tension-											-0.75	-1		≤	0
		Sliding+										1	-0.6			≤	0
		Sliding-										-1	-0.6			≤	0
		Objective	0	0	0	0	0	0	0	0	0	0	0	0	1		

Table A.1: Problem matrix ('Linear Programming tableau')

This problem can now be solved using any suitable linear programming solver. For this problem the critical load factor may be found to be 2.742.

Once solved, values for the contact stress resultant variables (shear force; normal force; moment) also become available. Additionally, quantities which correspond to these in a work sense also become available (sliding displacement; opening displacement; rotation). This allows the collapse mechanism to be identified, which in this case is a four hinge failure mechanism.

© *LimitState Ltd*

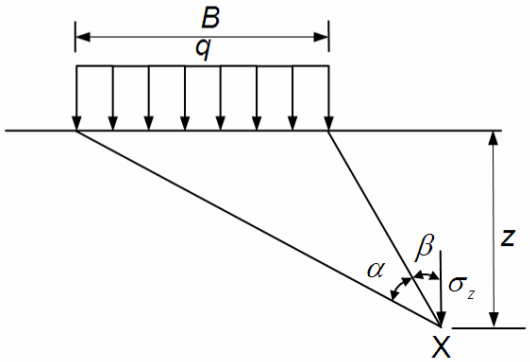
© *LimitState Ltd*

Appendix B

Additional notes on the backfill model

B.1 Boussinesq distribution model

According to Boussinesq theory, the vertical stress at point X, due to a uniform pressure q on a strip area of width B and infinite length, is given in terms of the angles and as defined in Figure B.1.



$$\sigma_z = \frac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha + 2\beta)\right]$$

Figure B.1: Vertical stresses calculated according to Boussinesq theory

Although the distribution has been used for many years in masonry arch analysis programs (e.g. Choo et al. 1991), from a theoretical perspective the use of a Boussinesq type distribution is not entirely satisfactory, since:

1. there does not exist a semi-infinite elastic half space below the load;
2. the elastic distribution indicated is not really compatible with an ultimate load analysis.

However, notwithstanding the above comments, the above equation does provide a useful means of generating a suitable 'bell shaped' curve.

Furthermore, to avoid excessive distribution at the ultimate limit state, when concentration of the load is likely, the Boussinesq distribution is truncated in LimitState:RING, with the computed

stresses factored up to ensure the vertical applied load is equal to the sum of the distributed load stresses multiplied by the areas over which they act. However, as applied loads are anyway normally placed at numerous positions across the bridge, no attempt is made to ensure that the centre of the line of action of an applied load coincides with the centre of the line of action of the distributed loads (i.e. moment equilibrium is not enforced).

B.2 Limiting horizontal fill stresses

The horizontal soil stresses applied to the extrados of a given voussoir are limited in Limit-State:RING to those which would just cause sliding of the overlying strip of soil.

The relevant vertical and horizontal stresses and forces, together with the normal and shear forces, are shown on Figure B.2.

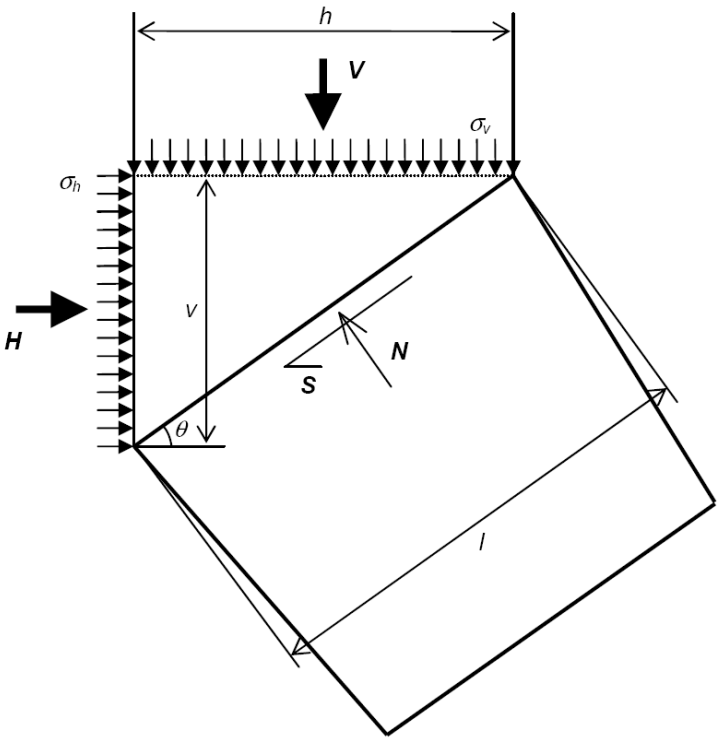


Figure B.2: Forces acting on extrados of a block subject to backfill pressures

If the specified soil / arch interface has friction δ and adhesion a, then the overlying strip of soil will just slide when:

$$S_{max} = N \tan \delta + al \tag{B.1}$$

Where:

$$S_{max} = H_{max} \cos \theta - V \sin \theta \tag{B.2}$$

© LimitState Ltd

$$N = H_{max} \sin\theta + V \cos\theta \tag{B.3}$$

Hence the maximum applied horizontal force can be expressed as:

$$H_{max} = \frac{al + V(\cos\theta \tan\delta + \sin\theta)}{\cos\theta - \sin\theta \tan\delta} \tag{B.4}$$

Now:

$$H_{max} = v\sigma_{h,max} \tag{B.5}$$

$$V = h\sigma_v \tag{B.6}$$

And hence the maximum horizontal stress that can be applied is:

$$\sigma_{h,max} = \frac{al + h\sigma_v(\cos\theta \tan\delta + \sin\theta)}{v(\cos\theta - \sin\theta \tan\delta)} = \frac{a + \cos\theta\sigma_v(\cos\sigma \tan\delta + \sin\theta)}{\sin\theta(cos\theta - \sin\theta \tan\delta)} \tag{B.7}$$

B.3 Passive and active fill pressures

At collapse, portions of the arch will move into the backfill, thereby mobilizing passive earth pressures. Other portions of the arch may move away from the soil, thereby mobilizing active earth pressures. The passive pressures in particular can have a significant effect on the arch collapse load. The semi-empirical soil model employed in LimitState:RING models passive earth pressures by applying empirical correction factors m_p and m_{pc} to the lateral earth pressure coefficients K_p and K_{pc} that are normally computed for vertical smooth retaining wall as follows:

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi} = \tan^2\left(45 + \frac{\phi}{2}\right) \tag{B.8}$$

$$K_{pc} = 2\sqrt{K_p} \tag{B.9}$$

The LimitState:RING soil model does not by default model active pressures. It is important to appreciate that quantitative values of m_p and m_{pc} are at present empirical and have limited theoretical basis. They correct for a number of effects including:

- the curved shape of the arch;
- the magnitude of the soil / arch interface friction and / or adhesion;
- gross displacement and strength mobilization effects (see also Appendix B.4);

© *LimitState Ltd*

- in-situ lateral earth pressures arising, for example, from compaction of the fill; and,
- active pressure effects on other portions of the arch.

Note that while values of the soil / arch interface properties can be specified in LimitState:RING, these are used in a limited way as discussed in Appendix B.2. In LimitState:RING, the default value of m_p is taken as 0.33. This has been shown to give a reasonable prediction of collapse load in physical model tests on single span arches. There is very limited data currently available for the effects of cohesion on arch collapse load. The default factor m_{pc} is therefore currently set conservatively at 0.05. (This has been increased from the default value of 0.01 utilised in LimitState:RING 2.0 in the light of additional experimental and numerical data.)

When ϕ is small, $m_p K_p$ can fall significantly below 1.0 for a value of m_p equal, for example, to the default 0.33. It may be argued that a value less than 1.0 indicates that the soil is not positively mobilising any friction to resist arch movement and is over conservative. There is therefore an option in LimitState:RING to ensure that the resultant value of $m_p K_p$ never falls below 1.0. This is on by default. Specific situations where this may be invalid include:

- situations involving small arch movements where the at rest (starting) lateral earth pressure coefficient K_0 was initially very much less than 1.0. This might occur in the unlikely event of poorly compacted backfill;
- situations involving low strength backfills where the active earth pressures are relatively high compared to the passive earth pressures (m_p corrects for several effects including active earth pressures as listed above). This is unlikely to occur for single span arches where an external load is applied above the active side such that the effect of the distributed load itself normally dominates the active side soil pressures. It may however be relevant for a deforming arch that is part of a multispan arch bridge, but not subject to an external load.

Burroughs et al. (2002) suggested using an alternative pressure coefficient for arches:

$$K_e = K_0 + e(K_p - K_0) \tag{B.10}$$

Where K_0 is the at rest earth pressure coefficient, typically taken as $1 - \sin\phi$ (though if the fill has been compacted then K_0 can take on significantly higher values), and e is an empirical factor. When ϕ is high and if e is taken as equal to e.g. the default value of m_p = 0.33, then the pressure coefficient $K_e \cong m_p K_p$ (since K_0 is very small compared with K_p). There is thus very little difference between this approach and that used by LimitState:RING.

For low values of ϕ, the main potential advantage of the approach proposed by Burroughs et al. is that it limits the minimum value of K_e to K_0. However since m_p is designed to correct for a number of factors, not just the simple passive pressure coefficient, it was considered inappropriate to use this as the default approach in LimitState:RING. However, since the user is free to select m_p, then this approach can be applied if desired by setting $m_p = K_e/K_p$. However for the same reasons discussed earlier with respect to the option 'Keep $m_p K_p$>=1.0', at the present time it is recommended that Burroughs' coefficient K_e is used with caution in LimitState:RING.

© LimitState Ltd

B.4 Gradual build-up of passive pressures

Experimental evidence has shown that peak passive pressures are only mobilised when structural deformations of sections of an arch into surrounding fill material are relatively large. It can therefore be argued that a gross displacement analysis (Gilbert 1997) is required to identify the peak load, and that a normal LimitState:RING analysis which assumes infinitesimal deformations is inappropriate.

However, in reality the fill in most short span bridges will be very well compacted (due to trafficking) and hence relatively large pressures can be expected to be mobilised even when displacements are relatively small. For example, Figure B.3 shows the mean horizontal pressures mobilised above the springings or piers of three of the most well instrumented bridges tested at Bolton in the 1990s (refer also to Appendix G). These bridges were backfilled with a well compacted graded crushed limestone fill material.

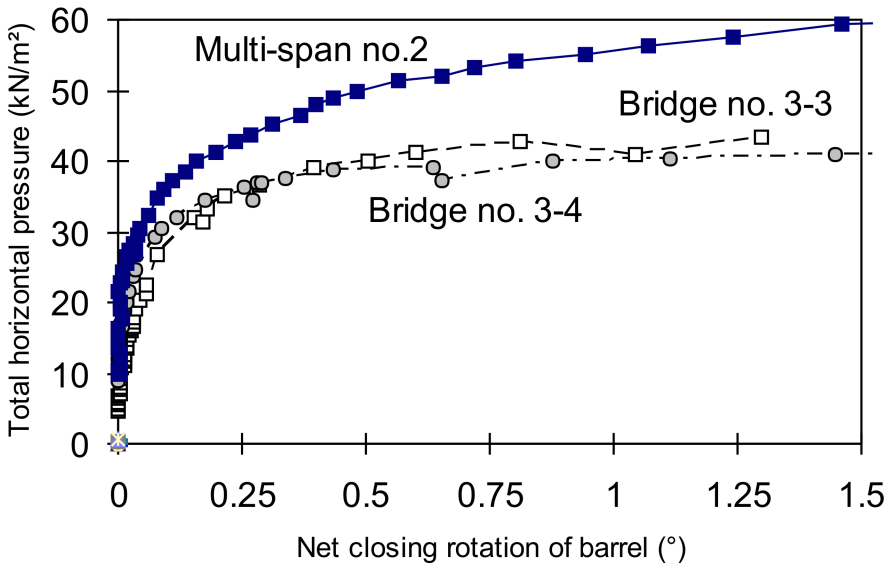


Figure B.3: Mean horizontal backfill pressure vs. net closing barrel rotation of arch barrels

A special version of LimitState:RING was developed to enable gross displacement analyses to be performed, with the experimentally observed build up in pressures back-substituted into the analysis. The trend shown in Figure B.4 was obtained.

© LimitState Ltd

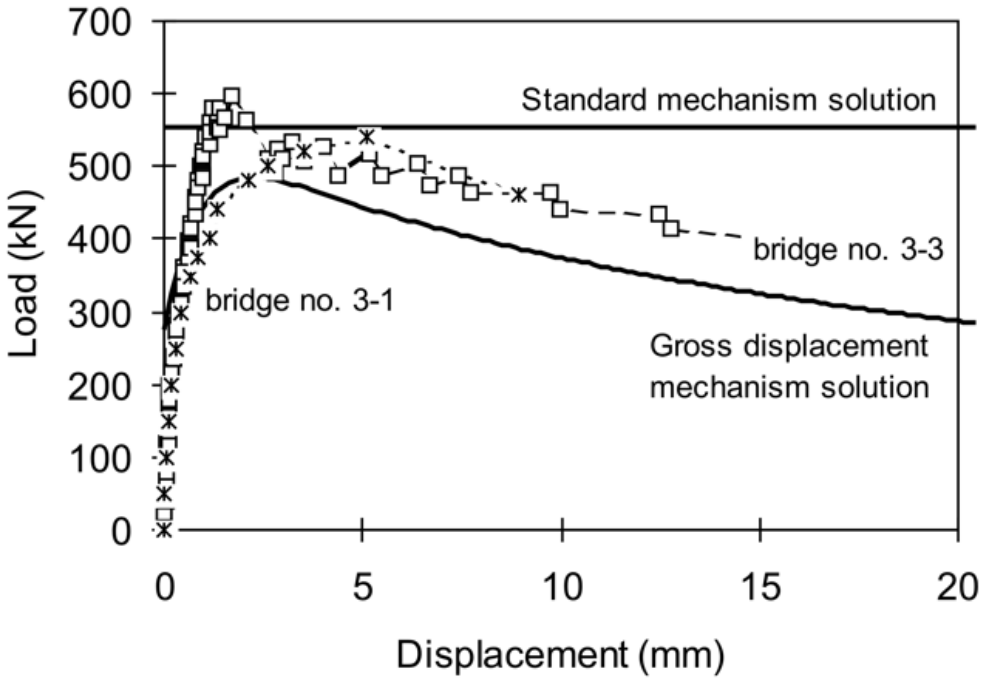


Figure B.4: Experimental and predicted load vs. displacement response of single-span bridges

Thus Figure B.4 indicates that the standard mechanism solution, which does not take gross structural movements into account, is in error by less than 15%. Had the build up in backfill pressures been more gradual then the error would have been greater. Nevertheless, given the numerous other uncertainties which exist when modelling masonry arch bridges, such errors may well be considered acceptable.

B.5 Unusual failure mechanisms

Most bridges tested in the laboratory to date have failed in 4 hinge failure mechanisms and, as indicated previously, taking $m_p = \frac{1}{3}$ (where the resultant coefficient of lateral earth pressure is calculated as $m_p K_p$) permits generally good predictions of carrying capacity to be obtained. However, it is currently unclear as to whether taking $m_p = \frac{1}{3}$ is applicable in all cases; LimitState:RING chooses the critical failure mechanism from a multitude of possible ones and a 4 hinge failure mechanism is by no means always identified as being critical.

For example, Figure 5.4(b) shows an alternative failure mode encountered when recently assessing a short span bridge. Here the predicted failure mode involves sliding failures at three joints, and translation rather than rotation of a section of arch into the fill. However, the magnitudes of the passive restraining pressures applied correspond to those mobilized in a 4 hinge mechanism. In reality the sliding failure mode predicted would be likely to more rapidly mobilize significant passive zone soil pressures. Thus the LimitState:RING strength prediction (using $m_p = \frac{1}{3}$) may be quite conservative.

© LimitState Ltd

In the future it is expected that this issue will be resolved by moving away from the current indirect modelling strategy for the soil towards instead modelling the soil explicitly (i.e. using solid elements to represent the soil material).

B.6 Backfill - Backwards compatibility with RING 1.5

The import facility in LimitState:RING will automatically convert RING 1.5 backfill settings to the correct equivalent values so that identical pressure distributions are modelled in LimitState:RING 3.2.a . The LimitState:RING 3.2.a model provides additional capabilities above those provided in RING 1.5. In order that users may manually replicate RING 1.5 settings in LimitState:RING 3.2.a and also understand how the import facility has represented RING 1.5 models, this section provides direct guidance on how to replicate RING 1.5 settings.

The RING 1.5 backfill properties tab is given in Figure B.5.

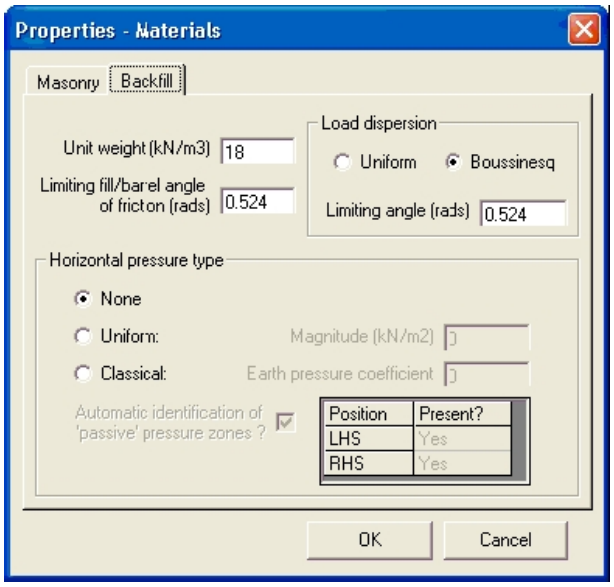


Figure B.5: RING 1.5 backfill properties tab

B.6.1 Unit weight

In LimitState:RING 3.2.a , enter this value under ‘Unit Weight’.

B.6.2 Limiting fill / barrel angle of friction

This requires an angle of friction ϕ to have been set in the LimitState:RING 3.2.a dialog (see the section on B.6.4) for guidance on setting this value. Let the required limiting backfill/arch barrel angle of friction be δ. In LimitState:RING 3.2.a , click on **Advanced** and in the section **Soil-arch interface properties** set the **Friction, multiplier on** ϕ value to δ/ϕ. Also set the **Adhesion, multiplier on** c value to zero.

B.6.3 Load dispersion type

In LimitState:RING 3.2.a , click on **Advanced**. The settings in the section 'Live load dispersion details' are directly equivalent to the RING 1.5 settings. Note that **cut off angle (deg)** in LimitState:RING 3.2.a is directly equivalent to **Limiting angle (rads)** in RING 1.5, except that the units are now degrees not radians.

B.6.4 Horizontal pressure type

None

To replicate this option in LimitState:RING 3.2.a , set ϕ and c to zero in the section **Soil Properties**. (Note the LimitState:RING 3.2.a import facility will also set the **Factor** m_p and **Factor** m_{pc} to zero, and uncheck the box **Keep** $m_p K_p >= 1.0$ in the **Advanced** section **Passive zone parameters**.)

Uniform

To replicate this option in LimitState:RING 3.2.a , in the section **Soil Properties**, set ϕ equal to δ (the selected value of the Limiting fill/barrel angle of friction) and c to the specified RING 1.5 uniform pressure **Magnitude**. Click on **Advanced** and in the section **Passive zone parameters** set the **Factor** m_p to zero and **Factor** m_{pc} to $(1/(2\tan(45+\delta/2)))$. Finally uncheck the box **Keep** $m_p K_p >= 1.0$. The value given in the box $m_{pc} K_{pc} c$ should be the required value of the uniform pressure. To correctly set the required limiting backfill/arch barrel angle of friction, set the **Friction, multiplier on** ϕ value to 1.0. Also set the **Adhesion, multiplier on** c value to zero.

Classical

To replicate this option in LimitState:RING 3.2.a , . First click on **Advanced** and in the section **Passive zone parameters** set the **Factor** m_p to a preferred value. This may be chosen as any value $0 \leq m_p \leq 1.0$. (The LimitState:RING 3.2.a import facility use the default value of 0.33.) Then set the **Factor** m_{pc} to zero and uncheck the box **Keep** $m_p K_p >= 1.0$.

Let the specified RING 1.5 **Earth Pressure Coefficient** be E. Compute the equivalent soil angle of friction ϕ using the following formula:

$$\phi = 2\left(\arctan\left(\sqrt{\frac{E}{m_p}}\right) - 45^o\right) \tag{B.11}$$

In the section **Soil Properties**, set ϕ to the value calculated in equation B.11 and c to zero. The value given in the box $m_p K_p$ should be the required value of E. Ensure that the correct procedure for replicating the RING 1.5 **Limiting fill/barrel angle of friction** value has been followed using the value of ϕ computed here.

© *LimitState Ltd*

B.6.5 Automatic identification of passive zones

This is implemented in the same way in LimitState:RING 3.2.a as in RING 1.5. Click 'Advanced' in LimitState:RING 3.2.a . The corresponding check box and table may be found in the section **Passive zone parameters**.

© *LimitState Ltd*

© *LimitState Ltd*

Appendix C

Default parameters

C.1 General

Parameter	**Default**	***Notes***
Bridge width	2500mm	*'Specified' width selected by default*

Table C.1: General default properties

C.2 Geometry

Parameter	**Default**	***Notes***
Intrados shape	Segmental	*Common in practice*
Number of units per ring	40	
Number of rings	1	*Most European bridges do not use multi-ring construction*
Depth of surface fill / ballast	500mm	*Default surface profile assumed to be level. Height of surface fill layer = y distance between left-hand intrados springing and base of surface fill layer.*
Height of surface fill layer	2000mm	

Table C.2: Geometry default properties

C.3 Transverse properties (only used with auto-computed bridge width)

Parameter	**Default**	***Notes***
Transverse sleeper length	2400mm	*Railway underline bridges only*
Transverse axle spacing	1800mm	*Highway bridges only*
Auto-computed extra width	a) 0mm b) 1500mm	*a) Railway bridges UIC 774-2R* *b) Highway bridges UK Highways Agency Standard BD21/01*
Transverse angle of dispersion of live loads (ballast / surface fill)	a) 15° b) 26.6°	*a) Railway bridges UIC 774-2R* *b) Highway bridges - equivalent to 2:1, as recommended in UK Highways Agency Standard BD21/01*
Transverse angle of dispersion of live loads (backfill)	a) 30° b) 26.6°	*a) Railway bridges* *b) Highway bridges - equivalent to 2:1, as recommended in UK Highways Agency Standard BD21/01*

Table C.3: Transverse default properties (only used with auto-computed bridge width)

© LimitState Ltd

C.4 Material properties

Parameter	**Default**	***Notes***
Unit weight (masonry)	20kN/m^3	
Coefficient of friction (radial)	0.6	*From BS5628(i)*
Coefficient of friction (tangential)	0.5	*Based on measured in Bolton laboratory bridges (mean measured value was 0.53) (Melbourne & Gilbert 1995)*
Masonry crushing strength	5N/mm^2	
Unit weight (surface fill / ballast)	18kN/m^3	
Unit weight (backfill)	18kN/m^3	
Longitudinal angle of dispersion of live loads (ballast / surface fill)	a) 15° b) 26.6°	*a) Railway bridges - UIC 774-2R* *b) Highway bridges - equivalent to 2:1, as recommended in UK Highways Agency Standard BD21/01*
Longitudinal angle of dispersion of live loads (backfill)	a) 30° b) 30°	*a) Railway bridges, b) Highway bridges* *LimitState:RING standard value, calibrated against load tests*
Angle of friction, ϕ	30°	
Cohesion, c	0 kN/m^2	
Model dispersion of live load	true	
Model horizontal 'passive' pressures	true	
Soil-arch interface, friction multiplier on ϕ	0.66	
Soil-arch interface, adhesion multiplier on c	0.5	
Fill dispersion model (ballast / surface fill)	Normal	
Fill dispersion model (backfill)	Boussinesq	*With 30° cutoff*
Passive zone factor m_p	0.33	
Passive zone factor m_{pc}	0.05	
Keep $m_p K_p > 1.0$	true	
Auto-identify passive zones	true	

Table C.4: Default material properties

C.5 Partial factors

Partial factor on	**Symbol**	***Default***
Permanent load, masonry	$\gamma_{f,m}$	*1.0*
Permanent load, fill	$\gamma_{f,f}$	*1.0*
Permanent load, surface fill / ballast	$\gamma_{f,sf}$	*1.0*
Permanent load, track	$\gamma_{f,t}$	*1.0*
Axle load	$\gamma_{f,l}$	*1.0*
Dynamic load	$\gamma_{f,dyn}$	*1.0*
Material (masonry strength)	$\gamma_{m,ms}$	*1.0*
Material (masonry friction)	$\gamma_{m,mf}$	*1.0*

Table C.5: Default partial factors

C.6 Track parameters

Parameter	**Default**	***Notes***
Sleeper spacing	500mm	*Conservative value, less than standard 600mm*
Sleeper breadth	250mm	*Standard (see Network Rail 2006)*
Sleeper length (transverse)	2400mm	*Standard sleeper length*
Track weight (incl. sleepers, rails and ballast between sleepers)	2.0kN/m^2	

Table C.6: Default track properties

© *LimitState Ltd*

Appendix D

Standard loading models

D.1 Railway loading models

D.1.1 UIC

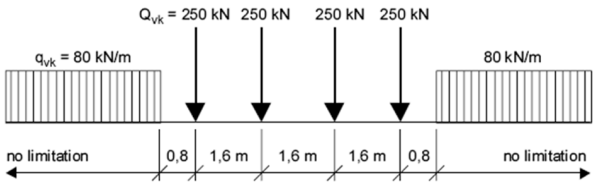
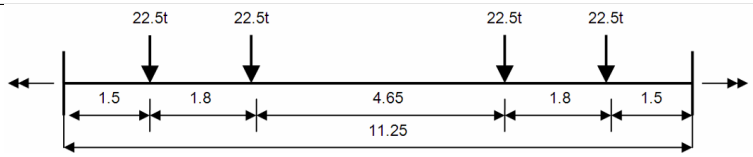
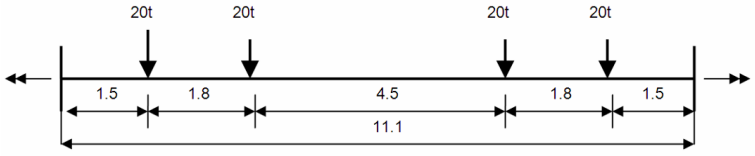
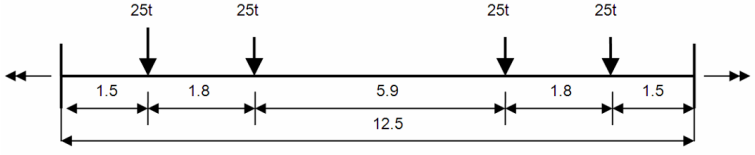
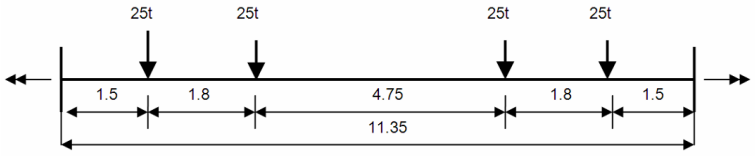
Description	Loading
Load model 71 - No UDL (UIC776-1R, UIC702)	 *distributed load components are not included in the LimitState:RING database
Load train D4 (UIC700)	
Load train C3 (UIC700)	
Load train E4 (UIC700)	
Load train E5 (UIC700)	 *distributed load components are not included in the LimitState:RING database

Table D.1: Standard UIC railway loading in LimitState:RING

© *LimitState Ltd*

D.1.2 BD37

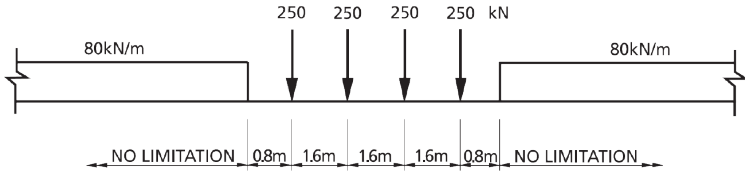
Description	Loading
RU loading, no UDL	 *distributed load components are not included in the LimitState:RING database
RL loading, no UDL (single 200kN axle)	No image in code
RL loading, no UDL (300kN & 150kN axles)	No image in code

Table D.2: Standard BD37 railway loading in LimitState:RING

D.1.3 Network Rail (NR/GN/CIV/025)

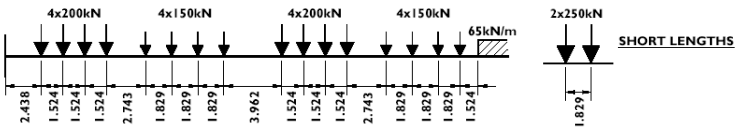
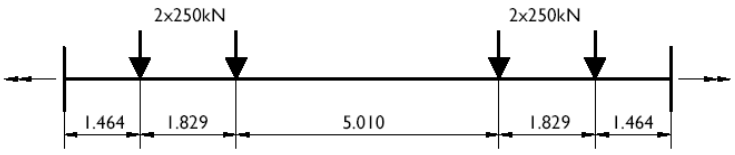
Description	Loading
Network Rail RA1 load train [20 units shown] (NR/GN/CIV/025)	 *distributed load components are not included in the LimitState:RING database
Network Rail Assessment Load Wagon (NR/GN/CIV/025)	

Table D.3: Standard Network Rail railway loading in LimitState:RING

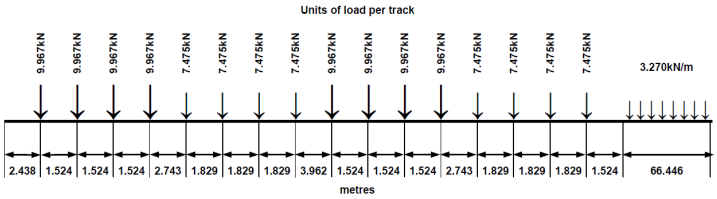
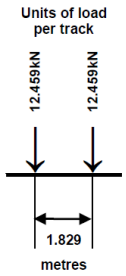
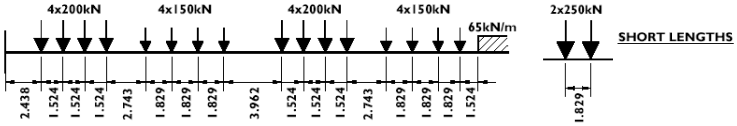
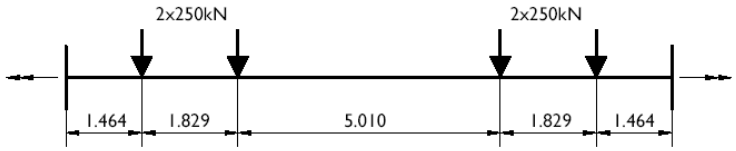
Description	Loading
Type RA1 Load, 1 BSU, No UDL	 *distributed load components are not included in the LimitState:RING database
Type RA1 Short Lengths, 1 BSU, No UDL	
Type RA1 Load, 20 BSU, No UDL (RA10)	 *distributed load components are not included in the LimitState:RING database
Type RA1 Short Lengths, 20 BSU, No UDL (RA10)	See above
Assessment Load Wagon	

Table D.4: Standard Network Rail railway loading in LimitState:RING

D.1.4 Indian Railways

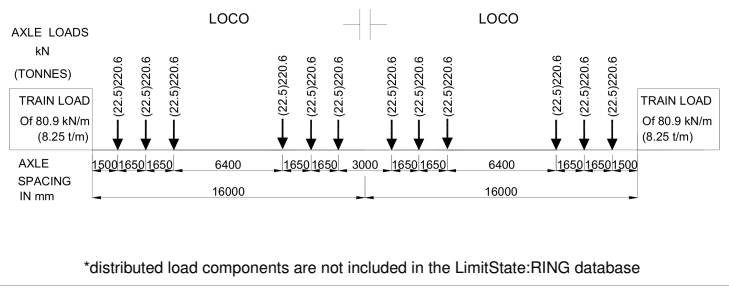
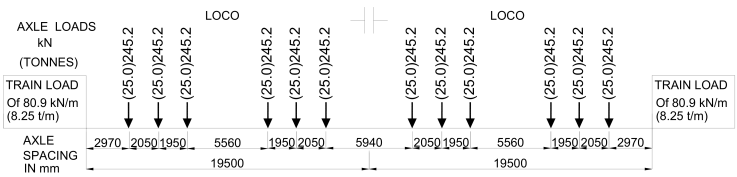
Description	Loading
Indian Railways Modified Broad Gauge Loading 1987 (A)	 *distributed load components are not included in the LimitState:RING database
Indian Railways Modified Broad Gauge Loading 1987 (B)	 *distributed load components are not included in the LimitState:RING database

Table D.5: Standard Indian Railways loading in LimitState:RING

D.2 Highway loading models

D.2.1 Construction and use

VEHICLE	AXLE WEIGHTS AND SPACING								
	W1 (kN)	A1 (m)	W2 (kN)	A2 (m)	W3 (kN)	A3 (m)	W4 (kN)	A4 (m)	W5 (kN)
Single Axle (10.5 Tonne)	103.01								
CU Double Axle (1.02m)	79.76	1.02	79.76						
CU Double Axle (1.85m)	99.77	1.30	99.77						
CU Triple Axle (1.40m)	58.86	0.70	58.86	0.70	58.86				
CU Triple Axle (2.70m)	73.58	1.35	73.58	1.35	73.58				
CU Triple Air Suspended Axle (2.60m)	78.48	1.30	78.48	1.30	78.48				
CU B1 (30 Tonne, 4-Axle Rigid)	59.74	1.02	59.74	4.08	89.76	1.20	89.76		
CU C1 (30 Tonne, 4-Axle Rigid)	54.74	1.20	54.74	3.80	94.76	1.50	94.76		
CU D1 (32 Tonne, 4-Axle Articulated)	39.83	2.00	99.67	4.20	89.76	1.20	89.76		
CU E1 (32 Tonne, 5-Axle Articulated)	59.74	3.10	89.76	1.20	89.76	4.00	39.83	1.02	39.83
CU F1 (38 Tonne, 5-Axle Articulated)	56.94	2.40	95.16	4.20	73.58	1.35	73.58	1.35	73.58
CU G1 (38 Tonne, 5-Axle Articulated)	58.86	2.80	73.58	1.50	103.01	5.28	68.67	1.02	68.67
CU G1* (38 Tonne, 5-Axle Articulated)	58.86	2.80	103.01	1.50	73.58	5.28	68.67	1.02	68.67
CU H1 (38 Tonne, 5-Axle Articulated)	57.49	2.45	58.86	1.35	103.01	5.28	76.71	1.02	76.71
CU H1* (38 Tonne, 5-Axle Articulated)	57.49	2.45	103.01	1.35	58.86	5.28	76.71	1.02	76.71

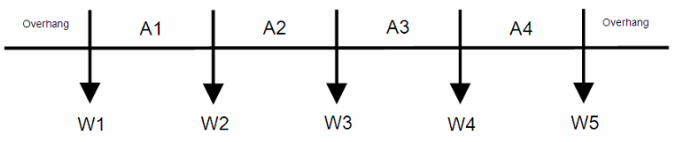


Table D.6: Construction and use. (Key: W1, W2 etc: axle weights (kN); A1, A2 etc: axle spacings (m); *axle weights reversed.)

D.2.2 Restricted construction and use

VEHICLE	AXLE WEIGHTS AND SPACING				
	W1 (kN)	A1 (m)	W2 (kN)	A2 (m)	W3 (kN)
RA (20 Tonne, 3-Axle Rigid)	39.83	2.18	79.76	1.02	79.76
RB (24 Tonne, 3-Axle Rigid)	59.84	3.70	89.66	1.20	89.66
RC (24 Tonne, 3-Axle Rigid)	59.84	3.60	99.67	1.40	79.76
RC* (24 Tonne, 3-Axle Rigid)	59.84	3.60	79.76	1.40	99.67
RD (24 Tonne, 3-Axle Rigid)	62.69	3.60	103.01	1.40	73.58
RD* (24 Tonne, 3-Axle Rigid)	62.69	3.60	73.58	1.40	103.01
RE (17 Tonne, 2-Axle Rigid)	63.77	3.00	103.01		
RF (7.5 Tonne, 2-Axle Rigid)	58.86	2.00	14.72		
RG (20 Tonne, 3-Axle Rigid)	20.60	2.00	8.83		

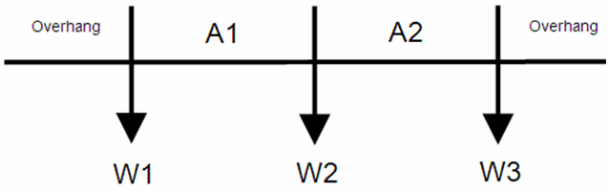


Table D.7: Restricted Construction and Use load vehicles. (Key: W1, W2 etc: axle weights (kN); A1, A2 etc: axle spacings (m); *axle weights reversed.)

© LimitState Ltd

D.2.3 European Union vehicles

VEHICLE	AXLE WEIGHTS AND SPACING										
	W1 (kN)	A1 (m)	W2 (kN)	A2 (m)	W3 (kN)	A3 (m)	W4 (kN)	A4 (m)	W5 (kN)	A5 (m)	W6 (kN)
Single Axle	112.82										
EU Double Axle (1.00m)	78.48	1.00	78.48								
EU Double Axle (1.30m)	88.29	1.30	88.29								
EU Double Axle Air (1.30m)	93.20	1.30	93.20								
EU Double Axle (1.80m)	98.10	1.80	98.10								
EU Triple Axle (2.60m)	68.67	1.30	68.67	1.30	68.67						
EU Triple Axle (2.80m)	78.48	1.40	78.48	1.40	78.48						
EU EC1 (26 Tonne, 3-Axle)	68.67	3.90	112.82	1.30	73.58						
EU EC1* (26 Tonne, 3-Axle)	68.67	3.90	73.58	1.30	112.82						
EU EC2 (32 Tonne, 4-Axle)	63.77	1.20	63.77	3.90	73.58	1.30	112.82				
EU EC2* (32 Tonne, 4-Axle)	63.77	1.20	63.77	3.90	112.82	1.30	73.58				
EU EC3 (40 Tonne, 5-Axle)	68.67	3.90	112.82	1.30	73.58	2.60	68.67	2.90	68.67		
EU EC3* (40 Tonne, 5-Axle)	68.67	3.90	73.58	1.30	112.82	2.60	68.67	2.90	68.67		
EU EC4 (44 Tonne, 6-Axle)	68.67	3.90	112.82	1.30	73.58	3.60	58.86	1.20	58.86	1.20	58.86
EU EC4* (44 Tonne, 6-Axle)	68.67	3.90	73.58	1.30	112.82	3.60	58.86	1.20	58.86	1.20	58.86

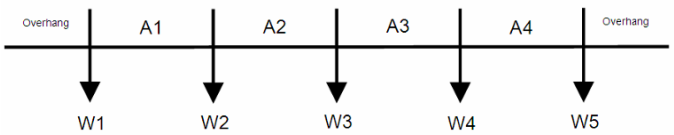


Table D.8: European Union load vehicles. (Key: W1, W2 etc: axle weights (kN); A1, A2 etc: axle spacings (m); *axle weights reversed.)

D.2.4 BD21 (Annex A) / AW (Schedule 3)

VEHICLE	AXLE WEIGHTS AND SPACING				
	W1 (kN)	A1 (m)	W2 (kN)	A2 (m)	W3 (kN)
11.5 Tonne Single Axle	112.82				
2x 8 Tonne Double Axle (1m Axle Spacing)	78.48	1.0	78.48		
2x 9 Tonne Double Axle (1.3m Axle Spacing)	88.29	1.3	88.29		
2x 9.5 Tonne Double Axle (1.3m Axle Spacing)	93.2	1.3	93.2		
Double Axle, 11.5 Tonne Driving (1.3m Axle Spacing)	112.82	1.3	73.56		
Double Axle, 10.5 Tonne Driving (1.3m Axle Spacing)	103.01	1.3	83.39		
2x 10 Tonne Double Axle (1.8m Axle Spacing)	98.10	1.8	98.10		
3x 7 Tonne Double Axle (1m Axle Spacing)	68.67	1.3	68.67	1.3	68.67
3x 8 Tonne Double Axle (1m Axle Spacing)	78.48	1.4	78.48	1.4	78.48

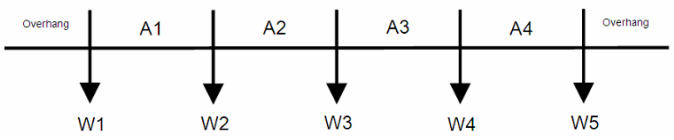


Table D.9: BD21 (Annex A) / AW (Schedule 3) load vehicles (Key: W1, W2 etc: axle weights (kN); A1, A2 etc: axle spacings (m); *axle weights reversed.)

© LimitState Ltd

D.2.5 BD21 (Annex D)

VEHICLE	AXLE WEIGHTS AND SPACING												
	O1 (m)	W1 (kN)	A1 (m)	W2 (kN)	A2 (m)	W3 (kN)	A3 (m)	W4 (kN)	A4 (m)	W5 (kN)	A5 (m)	W6 (kN)	O2 (m)
32 tonne, 4-Axle Rigid	1.00	63.77	1.20	63.77	3.90	112.82	1.30	73.58					1.00
38 tonne, 4-Axle, 2+2 Artic	1.00	63.77	3.00	112.82	5.10	98.10	1.80	98.10					1.00
40 tonne, 5-Axle, 2+3 Arctic	1.00	58.86	3.00	112.82	4.20	73.58	1.35	73.58	1.35	73.58			1.00
40 tonne, 5-Axle, 3+2 Artic	1.00	58.86	2.80	112.82	1.30	63.77	5.28	78.48	1.02	78.48			1.00
40 tonne, 5-Axle, 3+2 Artic*	1.00	58.86	2.80	63.77	1.30	112.82	5.28	78.48	1.02	78.48			1.00
40 tonne, 5-Axle, 3+2 Artic (10.5 tonne drive axle)	1.00	49.05	2.80	103.01	1.30	44.15	4.80	98.10	1.80	98.10			1.00
40 tonne, 5-Axle, 3+2 Artic (10.5 tonne drive axle)*	1.00	49.05	2.80	44.15	1.30	103.01	4.80	98.10	1.80	98.10			1.00
41 tonne, 6-Axle, 3+3 Artic	1.00	49.05	2.80	103.01	1.30	49.05	4.18	67.00	1.35	67.00	1.35	67.00	1.00
41 tonne, 6-Axle, 3+3 Artic*	1.00	49.05	2.80	49.05	1.30	103.01	4.18	67.00	1.35	67.00	1.35	67.00	1.00
44 tonne, 6-Axle, 3+3 Artic, (maximum axle weight 10.5 tonnes)	1.00	58.86	2.80	103.01	1.30	49.05	4.70	73.58	1.35	73.58	1.35	73.58	1.00
44 tonne, 6-Axle, 3+3 Artic, (maximum axle weight 10.5 tonnes)*	1.00	58.86	2.80	49.05	1.30	103.01	4.70	73.58	1.35	73.58	1.35	73.58	1.00
44 tonne, 5-Axle, 3+2 Artic (40ft ISO container)	1.00	68.67	2.80	112.82	1.30	73.58	7.60	88.29	1.35	88.29			1.00
44 tonne, 5-Axle, 3+2 Artic (40ft ISO container)*	1.00	68.67	2.80	73.58	1.30	112.82	7.60	88.29	1.35	88.29			1.00

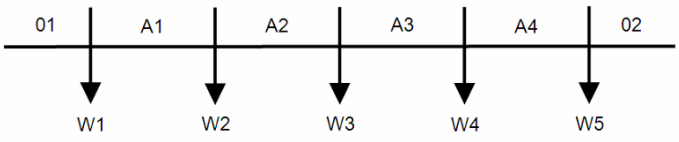


Table D.10: BD21 Annex D load vehicles. (Key: W1, W2 etc: axle weights (kN); A1, A2 etc: axle spacings (m); *axle weights reversed.)

D.2.6 BD21 (Annex E)

Make of Fire Engine	Gross Weight		Axle Spacing (m)	Weight Distribution (kN)		Group
	Tonnes	kN		Front	Rear	
Dennis DF	16.26	159.51	3.60	59.84	99.67	1
Dennis RS & SS	11.70	114.78	3.60	47.09	70.63	1
Dennis Rapier	11.00	107.91	3.60	43.95	63.95	1
Dennis Sabre	13.00	127.53	3.80	53.96	73.58	1
Dennis Sabre	14.50	142.25	4.20	53.96	88.29	1
Leyland MS 1600	16.26	159.51	3.68	64.84	99.77	1
Leyland MS 1600	16.26	159.51	4.62	64.84	99.77	1
Leyland MS 1600	16.26	159.51	5.26	64.84	99.77	1
Dodge	12.20	119.68	3.50	44.93	84.76	1
Dodge	13.21	129.59	3.80	47.38	89.76	1
Dodge	13.21	129.59	4.04	47.38	89.76	1
Dodge	16.26	159.51	4.50	64.84	99.77	1
Dodge	16.26	159.51	5.20	64.84	99.77	1
Dodge	16.26	159.51	5.80	64.84	99.77	1
Ford	13.00	127.53	3.73	47.38	89.76	1
Ford	13.00	127.53	4.04	47.38	89.76	1
Bedford SLR1	12.55	123.12	3.84	42.87	87.21	1
Bedford SLRA	12.55	123.12	3.51	42.87	87.21	1
Mercedes-Benz ATEGO	13.50	132.44	3.86	46.11	91.23	1
DAF FF55.230	16.26	159.51	3.90	44.15	93.20	1
Dodge	7.50	73.58	3.50	31.98	49.83	2
Dodge	6.60	64.75	3.60	22.56	48.07	2

Note: It is apparent that, for some vehicles, the summation of front and rear axle loads does not equal the stated gross weight. Values in the LimitState:RING vehicle database have been calculated directly from the values as given in the code and are therefore compliant with this document. It should also be noted that, in each case, the summation is always greater than the gross value and thus leads to the calculation of a conservative solution.

Table D.11: BD21 Annex E (Fire Engines)

© *LimitState Ltd*

D.2.7 BD21 (Annex F)

VEHICLE	AXLE WEIGHTS AND SPACING				
	W1 (kN)	A1 (m)	W2 (kN)	A2 (m)	W3 (kN)
11.5 Tonne Single Axle	112.82				
9 Tonne Single Axle	88.29				
7 Tonne Single Axle	68.67				
5.5 Tonne Single Axle	53.96				
3 Tonne Single Axle	29.43				
2x 9.5 Tonne Double Axle	93.20	1.3	93.20		
FE Group 1: 10 Tonne Single Axle	98.10				
FE Group 2: 5 Tonne Single Axle	49.05				

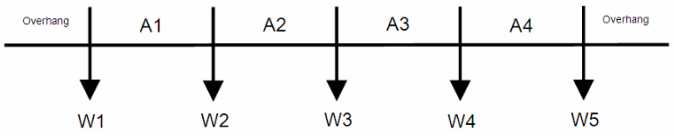


Table D.12: BD21 Annex F load vehicles (Key: W1, W2 etc: axle weights (kN); A1, A2 etc: axle spacings (m); *axle weights reversed.)

© *LimitState Ltd*

D.2.8 BD37 (HB Loading)

VEHICLE	AXLE WEIGHTS AND SPACING						
	W1 (kN)	A1 (m)	W2 (kN)	A2 (m)	W3 (kN)	A3 (m)	W4 (kN)
HB Load - 1 Unit (6m Inner Axle Spacing)	10	1.8	10	6.0	10	1.8	10
HB Load - 1 Unit (11m Inner Axle Spacing)	10	1.8	10	11.0	10	1.8	10
HB Load - 1 Unit (16m Inner Axle Spacing)	10	1.8	10	16.0	10	1.8	10
HB Load - 1 Unit (21m Inner Axle Spacing)	10	1.8	10	21.0	10	1.8	10
HB Load - 1 Unit (26m Inner Axle Spacing)	10	1.8	10	26.0	10	1.8	10
HB Load - 30 Units (6m Inner Axle Spacing)	300	1.8	300	6.0	300	1.8	300
HB Load - 30 Unit (11m Inner Axle Spacing)	300	1.8	300	11.0	300	1.8	300
HB Load - 30 Unit (16m Inner Axle Spacing)	300	1.8	300	16.0	300	1.8	300
HB Load - 30 Unit (21m Inner Axle Spacing)	300	1.8	300	21.0	300	1.8	300
HB Load - 30 Unit (26m Inner Axle Spacing)	300	1.8	300	26.0	300	1.8	300
HB Load - 37.5 Unit (6m Inner Axle Spacing)	375	1.8	375	6.0	10	1.8	375
HB Load - 37.5 Unit (6m Inner Axle Spacing)	375	1.8	375	11.0	10	1.8	375
HB Load - 37.5 Unit (6m Inner Axle Spacing)	375	1.8	375	16.0	10	1.8	375
HB Load - 37.5 Unit (6m Inner Axle Spacing)	375	1.8	375	21.0	10	1.8	375
HB Load - 37.5 Unit (6m Inner Axle Spacing)	375	1.8	375	26.0	10	1.8	375
HB Load - 45 Unit (6m Inner Axle Spacing)	450	1.8	450	6.0	450	1.8	450
HB Load - 45 Unit (6m Inner Axle Spacing)	450	1.8	450	11.0	450	1.8	450
HB Load - 45 Unit (6m Inner Axle Spacing)	450	1.8	450	16.0	450	1.8	450
HB Load - 45 Unit (6m Inner Axle Spacing)	450	1.8	450	21.0	450	1.8	450
HB Load - 45 Unit (6m Inner Axle Spacing)	450	1.8	450	26.0	450	1.8	450

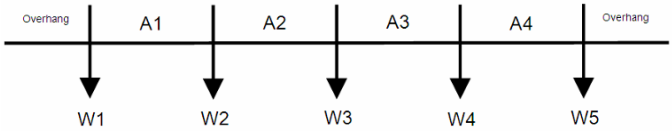
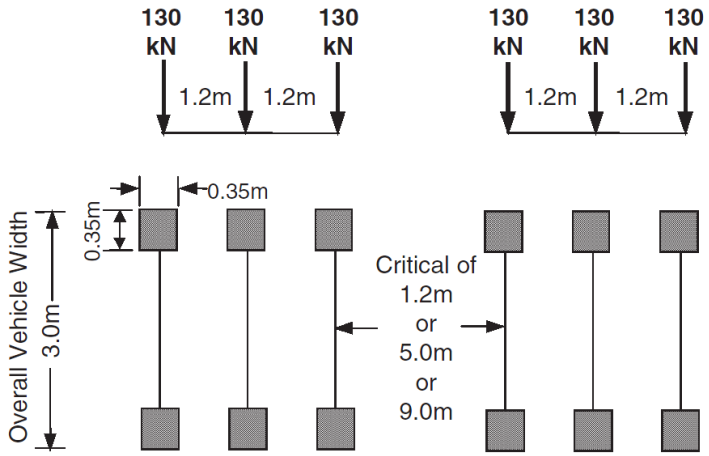


Table D.13: BD37 HB loading models. (Key: W1, W2 etc: axle weights (kN); A1, A2 etc: axle spacings (m))

D.2.9 BD86 Special Vehicles

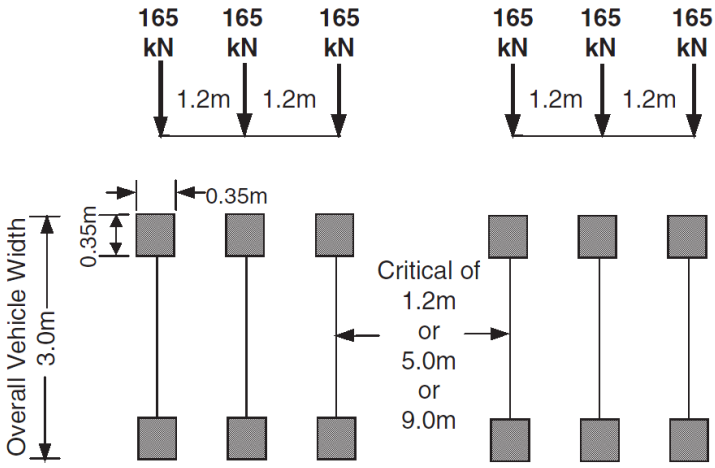
SV80



Note: Overall vehicle width = overall track width

Figure D.1: BD86 SV80 loading vehicle (all permutations included in LimitState:RING vehicle database).

SV100



Note: Overall vehicle width = overall track width

Figure D.2: BD86 SV100 loading vehicle (all permutations included in LimitState:RING vehicle database).

SV150

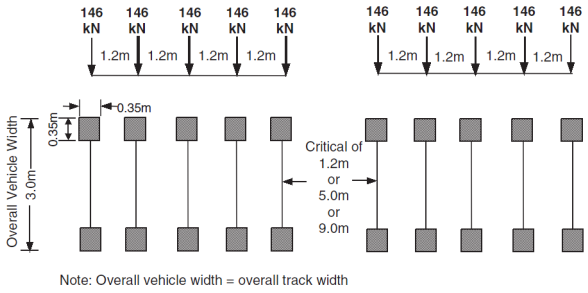


Figure D.3: BD86 SV150 loading vehicle (all permutations included in LimitState:RING vehicle database).

SV Train (SV196)

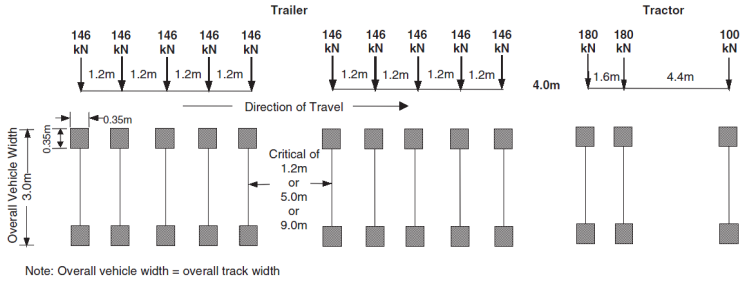


Figure D.4: BD86 SVTrain (SV 196) loading vehicle (all permutations included in LimitState:RING vehicle database).

SVTT

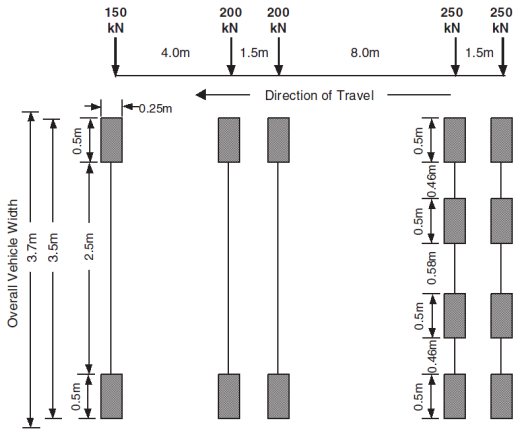


Figure D.5: BD86 SVTT loading vehicle (note direction of travel).

D.2.10 BD91

VEHICLE	AXLE WEIGHTS AND SPACING												
	O1 (m)	W1 (kN)	A1 (m)	W2 (kN)	A2 (m)	W3 (kN)	A3 (m)	W4 (kN)	A4 (m)	W5 (kN)	A5 (m)	W6 (kN)	O2 (m)
32 tonne, 4-Axle Rigid	1.00	63.77	1.20	63.77	3.90	112.82	1.30	73.58					1.00
38 tonne, 4-Axle, 2+2 Artic	1.00	63.77	3.00	112.82	5.10	98.10	1.80	98.10					1.00
40 tonne, 5-Axle, 2+3 Arctic	1.00	58.86	3.00	112.82	4.20	73.58	1.35	73.58	1.35	73.58			1.00
40 tonne, 5-Axle, 3+2 Artic	1.00	58.86	2.80	112.82	1.30	63.77	5.28	78.48	1.02	78.48			1.00
40 tonne, 5-Axle, 3+2 Artic*	1.00	58.86	2.80	63.77	1.30	112.82	5.28	78.48	1.02	78.48			1.00
40 tonne, 5-Axle, 3+2 Artic (10.5 tonne drive axle)	1.00	49.05	2.80	103.01	1.30	44.15	4.80	98.10	1.80	98.10			1.00
40 tonne, 5-Axle, 3+2 Artic (10.5 tonne drive axle)*	1.00	49.05	2.80	44.15	1.30	103.01	4.80	98.10	1.80	98.10			1.00
41 tonne, 6-Axle, 3+3 Artic	1.00	49.05	2.80	103.01	1.30	49.05	4.18	67.00	1.35	67.00	1.35	67.00	1.00
41 tonne, 6-Axle, 3+3 Artic*	1.00	49.05	2.80	49.05	1.30	103.01	4.18	67.00	1.35	67.00	1.35	67.00	1.00
44 tonne, 6-Axle, 3+3 Artic, (maximum axle weight 10.5 tonnes)	1.00	58.86	2.80	103.01	1.30	49.05	4.70	73.58	1.35	73.58	1.35	73.58	1.00
44 tonne, 6-Axle, 3+3 Artic, (maximum axle weight 10.5 tonnes)*	1.00	58.86	2.80	49.05	1.30	103.01	4.70	73.58	1.35	73.58	1.35	73.58	1.00
44 tonne, 5-Axle, 3+2 Artic (40ft ISO container)	1.00	68.67	2.80	112.82	1.30	73.58	7.60	88.29	1.35	88.29			1.00
44 tonne, 5-Axle, 3+2 Artic (40ft ISO container)*	1.00	68.67	2.80	73.58	1.30	112.82	7.60	88.29	1.35	88.29			1.00

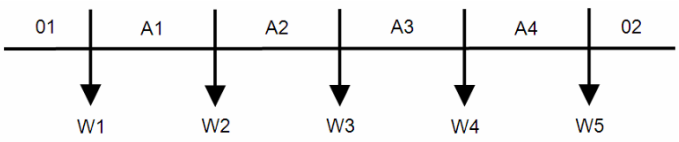


Table D.14: BD91 loading vehicles. (Key: W1, W2 etc: axle weights (kN); A1, A2 etc: axle spacings (m); *axle weights reversed.)

© LimitState Ltd

Appendix E

Worked examples - general

E.1 Example 1 - single span stone voussoir underline railway bridge

E.1.1 Details

Bridge name: Case study example 1[1]

Description: An initial LimitState:RING assessment of a single-span stone voussoir underline railway arch bridge is described. The bridge spans squarely between abutments and currently carries a single straight track. The bridge is in a fair condition, with:

- mortar loss of approx. 20mm on average in the vicinity of the arch springings;
- two longitudinal cracks in the arch barrel underneath the edges of the track, approx 3.1m apart.

Load model LM71 is used in this assessment.

Photograph:

[1] This is a fictitious bridge, with details taken from several real bridges.

Figure E.1: Photograph of Example 1 bridge

Commentary:

When undertaking an initial assessment of a bridge there will often be question-marks over certain dimensions, internal constructional details and material properties. A prudent strategy is to use best guess values for the relevant parameters initially but to subsequently undertake parametric studies to determine the sensitivity of the analysis to the assumptions made. This ensures that a subsequent detailed dimensional survey and/or intrusive investigation can focus in on the features of the bridge which have been identified to be most important.

Other issues specifically relevant to this bridge:

- The influence of the mortar loss should be considered separately to see whether re-pointing is an immediate priority.

- The presence of longitudinal cracks in the arch barrel will be likely to limit the effective bridge width (to 3.1m).

The first step in the assessment is to assemble the data necessary in order to undertake an analysis:

© *LimitState Ltd*

E.1.2 Assessment data

Parameter	**Value**	***Notes***
Span	5480mm	*Measured values (arch thickness confirmed by drillings at the crown and springings)*
Rise	2105mm	
Thickness	340mm	
Depth of fill & ballast below the underside of the sleepers above the crown	1155mm	
Effective width of bridge	3100mm	*Governed by the presence of longitudinal cracks in arch barrel*
Intrados shape	Segmental	*First approximation in the absence of a comprehensive dimensional survey*
Number of units per ring	23	*Actual number of voussoirs*
Number of rings	1	*Single ring of voussoirs*

Table E.1: Bridge geometry

Parameter	**Value**	***Notes***
Depth of ballast below sleeper	300mm	*Estimated value*
Sleeper breadth	250mm	*Typical value (timber sleeper)*
Sleeper length	2400mm	*Typical value (timber sleeper)*

Table E.2: Track geometry

Parameter	**Value**	***Notes***
Unit weight (masonry)	26 kN/m^3	*Estimated value (limestone)*
Coefficient of friction (radial)	0.6	*Estimated value*
Crushing strength	20 N/mm^2	*Estimated value (limestone blocks with thin joints $<$ 8mm)*
Unit weight (backfill)	18 kN/m^3	*Estimated value*
Angle of friction (backfill)	30^0	*Default value*
Cohesion (backfill)	0 kN/m^2	*Default value*
Unit weight (ballast)	18 kN/m^3	*Estimated value*
Track weight (incl. sleepers, rails and ballast between sleepers)	2.40kN/m^2	*Assuming 7.45kN/m length, taken from NR/GN/CIV/025 (assuming BH rail and timber sleepers)*

Table E.3: Material properties (characteristic values)

© LimitState Ltd

Parameter	**Value**	***Notes***
Load model name	LM71 (no udl)	*Distributed part of this load model can be ignored for this short span bridge*

Table E.4: Live load models to be considered

Parameter	**Value**	***Notes***
Spread through ballast (longitudinal)	15^0	*4:1 from UIC 774-2R / EN 1991-2*
Spread through ballast (transverse)	15^0	*4:1 from UIC 774-2R / EN 1991-2*
Spread through fill (transverse)	30^0	
Fill dispersion model (longitudinal)	Boussinesq (with 30^0 cutoff)	

Table E.5: Live load dispersion parameters

Parameter	**Value**	***Notes***
Masonry strength	2.5	
Masonry friction	1.0	
Masonry unit weight	0.95 (A)	*Also check with 1.35 (B)*
Fill unit weight	0.95 (A)	*Also check with 1.2 (B)*
Ballast unit weight	0.95 (A)	*Also check with 1.2 (B)*
Track load	0.95 (A)	*Also check with 1.2 (B)*
Axle load	1.0	
Dynamic	1.0	

Table E.6: Partial factors

E.1.3 Analysis results

Analysis 1: Partial factors 'A', reducing dead load effects

Computed adequacy factor = 2.44 (axles spaced between 3200 and 8000mm from the left springing). The associated failure mechanism involves 4 hinges:

© *LimitState Ltd*

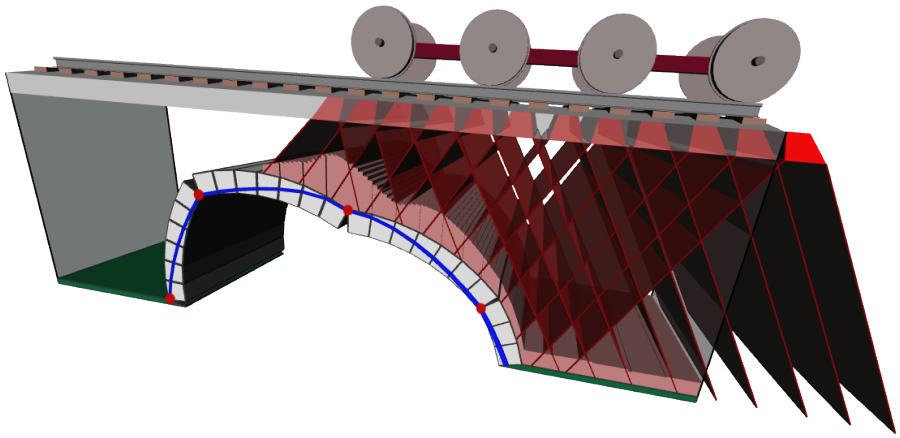


Figure E.2: Example 1: 4 hinge failure mechanism

Analysis 2: Mortar loss and partial factors 'A', reducing dead load effects

Computed adequacy factor = 2.44 (axles spaced between 3200 and 8000mm from the left springing). i.e. *30mm mortar loss near the springings (bottom two mortar joints) does not affect the computed load carrying capacity.*

Analysis 3: Partial factors 'B', increasing dead load effects

Computed adequacy factor = 2.98 (axles spaced between 3200 and 8000mm from the left springing). i.e. *this case is not critical.*

E.1.4 Next steps

- Dynamic factors can now be applied to the computed adequacy factor as deemed appropriate.
- The validity of the assumptions made should where possible be verified. e.g. the shape of the arch should be verified as this can have an important influence on the computed load carrying capacity.
- If the computed load carrying capacity proves to be insufficient then consideration should be given to carrying out a more detailed investigation of the fill and/or backing (if present).

© *LimitState Ltd*

E.2 Example 2 - multi-span, multi-ring brickwork underline railway bridge

E.2.1 Details

Bridge name: Case study example 2[2]

Description: An initial LimitState:RING assessment of a multi-span, multi-ring brickwork underline railway arch bridge is described. The bridge spans squarely between abutments and piers and currently carries two straight tracks. The 8 semicircular spans are nominally identical.

The bridge was constructed using engineering bricks and lime mortar and is in a reasonably good condition, with only isolated instances of loss of mortar from the joints and/or minor cracking. A hammer survey indicated no evidence of ring separation.

Initial intrusive investigations have indicated that the bridge contains solid piers. However intrusive investigations have not yet been undertaken to identify the extent / nature of any backing above the piers (although photographs show staining of the brickwork below what seems likely to be the top level of backing).

Load model LM71 is used in this assessment.

Photographs:

Figure E.3: Photographs of Example 2 bridge

Commentary:

- This bridge will initially be analysed assuming that the rings forming the arch barrel are

[2]This is a fictitious bridge, with details taken from several real bridges.

well bonded together; the influence of potential ring separation will then be considered to obtain a more conservative estimate of bridge strength.

- When all arches in a viaduct have nominally identical geometry, modelling only the two spans either side of the tallest pier is generally sufficient for the purposes of a preliminary assessment.

- In the preliminary assessment backing will conservatively be ignored (although parametric studies have indicated that this could enhance carrying capacity by around 30%-40%).

E.2.2 Assessment data

Parameter	**Value**	***Notes***
Effective width	3930mm	*Governed by the presence of longitudinal cracks in arch barrel*

Table E.7: General

Parameter	**Value**	***Notes***
Span	13610mm	*Survey values*
Rise	6780mm	
Thickness	682.5mm	
Depth of fill & ballast below the underside of the sleepers above the crown	1217.5mm	
Intrados shape	Segmental	*First approximation in the absence of a comprehensive dimensional survey*
Number of units per ring	80	*Large number to increase precision*
Number of rings	1 or 6	*6 rings in practice*

Table E.8: Bridge geometry: span 1

Parameter	**Value**	***Notes***
Height	6170mm	*Survey values*
Width (top)	2035mm	
Width (bottom)	2850mm	
Number of units in pier	20	

Table E.9: Bridge geometry: pier between span 1 and 2

© *LimitState Ltd*

Parameter	**Value**	***Notes***
Span	13610mm	*Measured values*
Rise	6780mm	
Thickness	682.5mm	
Depth of fill & ballast below the underside of the sleepers above the crown	1217.5mm	
Intrados shape	Segmental	*First approximation in the absence of a comprehensive dimensional survey*
Number of units per ring	80	*Large number to increase precision*
Number of rings	1 or 6	*6 rings in practice*

Table E.10: Bridge geometry: span 2

Parameter	**Value**	***Notes***
Depth of ballast below sleeper	300mm	*Estimated value*
Sleeper breadth	250mm	*Standard width*
Sleeper length	2400mm	

Table E.11: Track geometry

Parameter	**Value**	***Notes***
Unit weight (masonry)	20 kN/m^3	*Estimated value (engineering bricks)*
Coefficient of friction (radial)	0.6	*Typical value*
Coefficient of friction (tangential)	0.5	*Typical for degraded mortar (Bolton laboratory tests measured value)*
Crushing strength (masonry)	15 N/mm^2	*Estimated value (engineering bricks with thin joints $<$ 8mm)*
Unit weight (backfill)	21 kN/m^3	*Estimated value (most of fill actually brickwork backing)*
Angle of friction (backfill)	30^0	*Default value*
Cohesion (backfill)	0 kN/m^2	*Default value*
Unit weight (ballast)	18 kN/m^3	*Estimated value*
Track weight (incl. sleepers, rails and ballast between sleepers)	1.90 kN/m^2	*7.45kN/m length, taken from NR/GN/CIV/025 (assuming BH rail and timber sleepers)*

Table E.12: Material properties (characteristic values)

© LimitState Ltd

Parameter	**Value**	***Notes***
Load model name	LM71 (no udl)	*Distributed part of this load model ignored initially, but depending on observed failure mechanisms it may be necessary to include this later*

Table E.13: Live load models to be considered

Parameter	**Value**	***Notes***
Spread through ballast (longitudinal)	15^0	*4:1 from UIC 774-2R / EN 1991-2*
Spread through ballast (transverse)	15^0	*4:1 from UIC 774-2R / EN 1991-2 (not used)*
Spread through fill (transverse)	30^0	*(not used)*
Fill dispersion model (longitudinal)	Boussinesq (with 30^0 cutoff)	

Table E.14: Live load dispersion parameters

Parameter	**Value**	***Notes***
Masonry strength	2.5	
Masonry friction	1.0	
Masonry unit weight	1.35 (A)	*Also check with 0.95 (B)*
Fill unit weight	1.2 (A)	*Also check with 0.95 (B)*
Ballast unit weight	1.2 (A)	*Also check with 0.95 (B)*
Track load	1.2 (A)	*Also check with 0.95 (B)*
Axle load	1.0	
Dynamic load	1.0	

Table E.15: Partial factors

E.2.3 Analysis results

Analysis 1: Partial factors 'A', reducing dead load effects

Computed adequacy factor = 3.84 (axles spaced between 21200 and 26000mm from the far left springing). The associated failure mechanism involves both spans but not the intermediate pier:

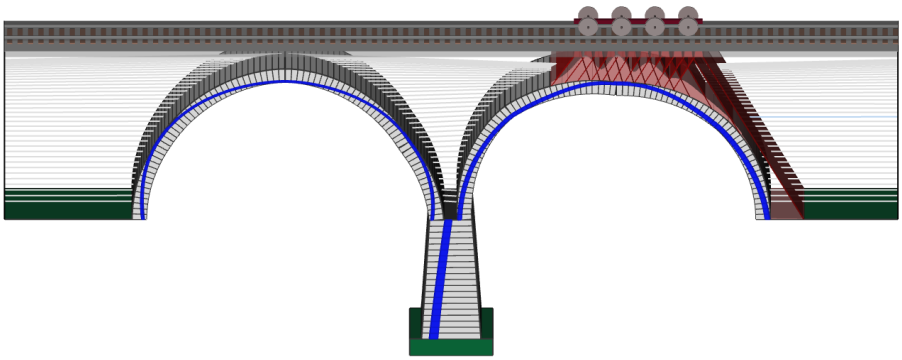


Figure E.4: Example 2: 2 span failure mechanism

Analysis 2: Partial factors 'B', reducing dead load effects and including separated rings

Computed adequacy factor = 1.11 (axles spaced between 3364 and 8164mm from the far left springing). The associated failure mechanism involves both spans (note the smooth deformed shapes of the two arches). The intermediate pier is not involved in the mechanism:

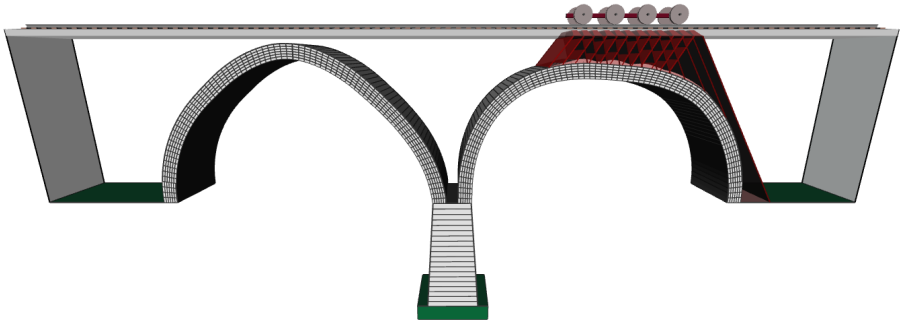


Figure E.5: Example 2: 2 span failure mechanism (multi-ring)

E.2.4 Next steps

- Clearly there is a very large gap between the computed failure loads arising from Analysis 1 and Analysis 2. Whilst the real failure load is likely to lie somewhere in between, this is in practice difficult to accurately determine (the assessment engineer may wish also to try including different amounts of ring separation in the analysis).
- Consideration should be given to including backing (or stronger fill material) in the analysis; this may help to compensate for the damaging effects of ring separation. The presence or otherwise of this backing material should preferably be later verified by carrying out appropriate intrusive investigations.
- Additionally:
  - Dynamic factors can be applied to the computed adequacy factor as deemed appropriate.

© *LimitState Ltd*

- Checks can be undertaken to ensure that different partial factors (increasing dead load effects) are not more onerous.
- LM71 should be applied with the addition of a user specified length of distributed load.
- For simplicity the adequacy factor has been calculated using the loading position found to be critical in the previous (single ring) analysis. Additional analyses should be performed to ensure the critical loading position has not changed.

© *LimitState Ltd*

© LimitState Ltd

Appendix F

Worked examples - reinforcement

(Note that in these examples the hand calculated values are very slightly different to those obtained using the software due to the requirement in the software to include a very small (1mm) camber.)

F.1 Case 1 - All reinforcement in full tension

In this example the depth of concrete crushing remains entirely in the area above the top reinforcement therefore both top and bottom reinforcement are in full tension at failure.

The beam properties are in Table F.1:

Property	**Value**
Block size	250mm x 1000mm (bridge width)
Beam span	5000mm - 100mm (block width) = 4900mm
Applied force	1kN @ span/2
Top reinforcement	100kN @ 50mm from top surface
Bottom reinforcement	100kN @ 50mm from bottom surface
Concrete crushing strength	5×10^{-3}kN/mm^2

Table F.1: Reinforced beam worked example 'Case 1' properties

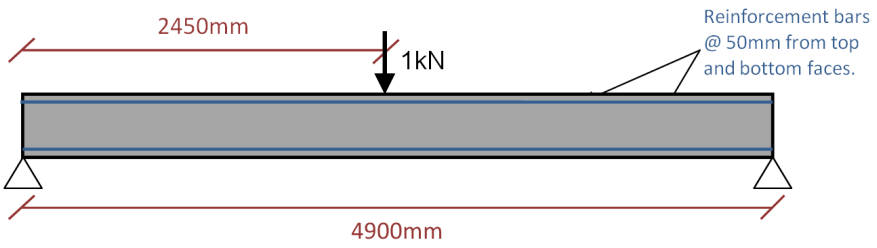


Figure F.1: Reinforced beam dimensions - Case 1

Applied moment

Applied Moment $=$ $\frac{\text{Applied force x Span}}{4}$ $=$ $\frac{1 \text{ x } 4900}{4}$ $=$ 1225kNmm

Initial assumed concrete force

Concrete force $=$ 2 x 100 $=$ 200kN

Concrete crushing depth

Concrete crushing depth = Concrete force / Bridge width / Concrete crushing strength

Concrete crushing depth = (200 / 1000) / (5×10^{-3}) = 40mm

Hence, the assumption that both the top and bottom reinforcement steel are in full tension is correct.

Moment capacity

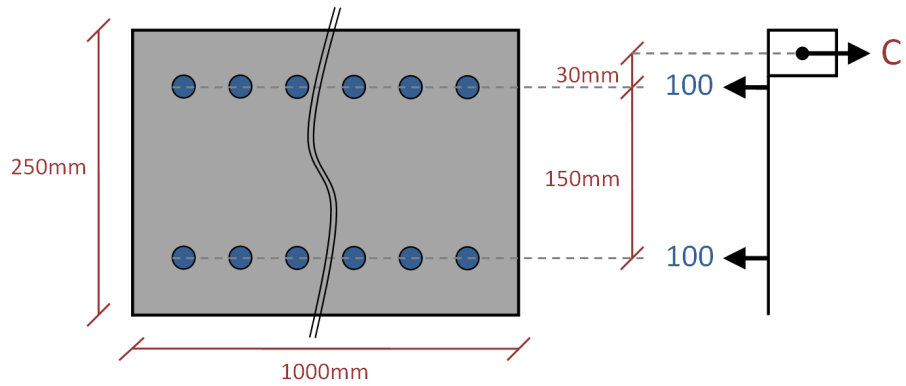


Figure F.2: Reinforced beam stress block - Case 1

Taking moments about centre of compression block (@ depth = 20mm):

Moment capacity = (30 x 100) + (180 x 100) = 21000kNmm

Adequacy factor

AF = Moment capacity / Applied moment = 21000 / 1225 = 17.1429

LimitState:RING calculated adequacy factor

AF = 17.1 (to 3 significant figures)

F.2 Case 2 - Bottom reinforcement in full tension, top reinforcement in partial tension

In this example the bottom reinforcement is in full tension and the top reinforcement carries a tensile force that must be determined.

The beam properties are shown in Table F.2.

Property	Value
Block size	250mm x 1000mm (bridge width)
Beam span	5000mm - 100mm (block width) = 4900mm
Applied force	1kN @ span/2
Top reinforcement	200kN @ 50mm from top surface
Bottom reinforcement	200kN @ 50mm from bottom surface
Concrete crushing strength	5×10^{-3}kN/mm^2

Table F.2: Reinforced beam worked example 'Case 2' properties

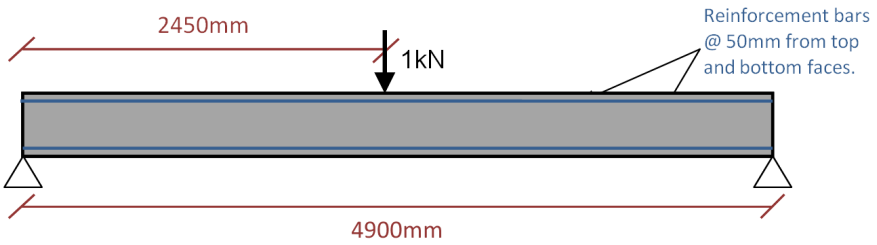


Figure F.3: Reinforced beam dimensions - Case 2

Applied moment

Applied Moment $= \frac{\text{Applied force x Span}}{4} = \frac{1 \text{ x } 4900}{4} =$ 1225kNmm

Initial assumed concrete force

Concrete force $=$ 2 x 200 $=$ 400kN

Concrete crushing depth

Concrete crushing depth = Concrete force / Bridge width / Concrete crushing strength

Concrete crushing depth = (400 / 1000) / (5×10^{-3}) = 80mm

Hence, the crushing depth is apparently greater than the depth to the top reinforcement bar which is clearly incorrect. To determine the force in the top bar and calculate the moment capacity of the section, we must calculate the force in the concrete and the steel:

Revised assumed concrete force

Concrete force = 50 x 1000 x 5 $\times$ 10^{-3} = 250kN

Top steel reinforcement force

Top steel force = 250 - 200 = 50kN

Hence both reinforcement bars are in tension, but the top bar is not fully yielding.

Moment capacity

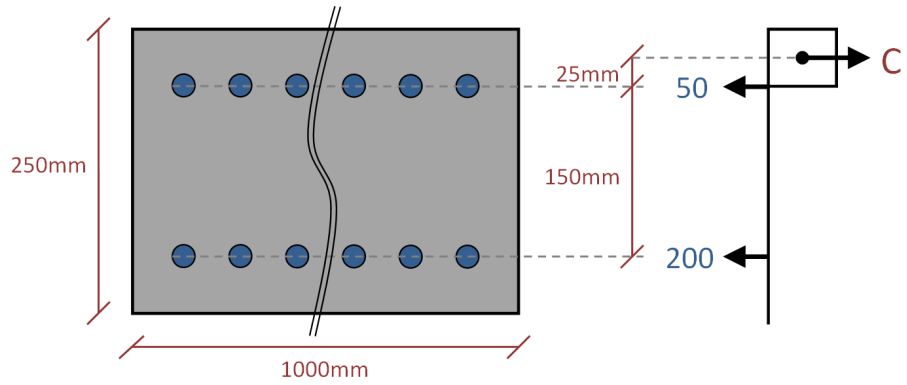


Figure F.4: Reinforced beam stress block - Case 2

Taking moments about centre of compression block (@ depth = 25mm):

Moment capacity = (25 x 50) + (175 x 200) = 36250kNmm

Adequacy factor

AF = Moment capacity / Applied moment = 36250 / 1225 = 29.5918

LimitState:RING calculated adequacy factor

AF = 29.6 (to 3 significant figures)

F.3 Case 3 - Bottom reinforcement in full tension, top reinforcement in full compression

In this example the concrete crushes to a depth below the top reinforcement, with both bars fully stressed (the top bar being in full compression and the bottom bar in full tension). The beam properties are given in Table F.3.

Property	Value
Block size	250mm x 1000mm (bridge width)
Beam span	5000mm - 100mm (block width) = 4900mm
Applied force	1kN @ span/2
Top reinforcement	100kN @ 50mm from top surface
Bottom reinforcement	400kN @ 50mm from bottom surface
Concrete crushing strength	5×10^{-3}kN/mm2

Table F.3: Reinforced beam worked example 'Case 2' properties

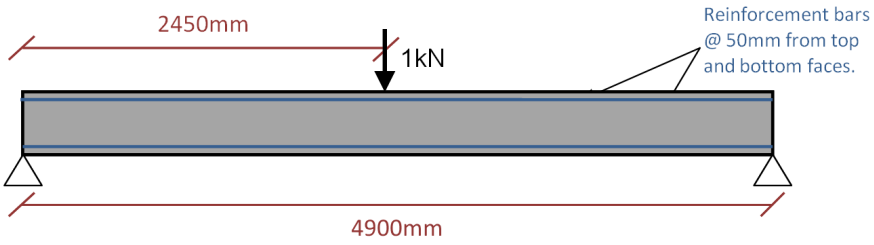


Figure F.5: Reinforced beam dimensions - Case 3

Applied moment

Applied Moment $= \frac{\text{Applied force x Span}}{4} = \frac{1 \text{ x } 4900}{4} =$ 1225kNmm

Initial assumed concrete force

Concrete force $=$ 400 - 100 $=$ 300kN

Concrete crushing depth

Concrete crushing depth = Concrete force / Bridge width / Concrete crushing strength

Concrete crushing depth = (300 / 1000) / (5×10^{-3}) = 60mm

Hence, the crushing depth is greater than the depth to the top reinforcement bar (i.e. bottom bar in full tension and top bar in full compression).

Moment capacity

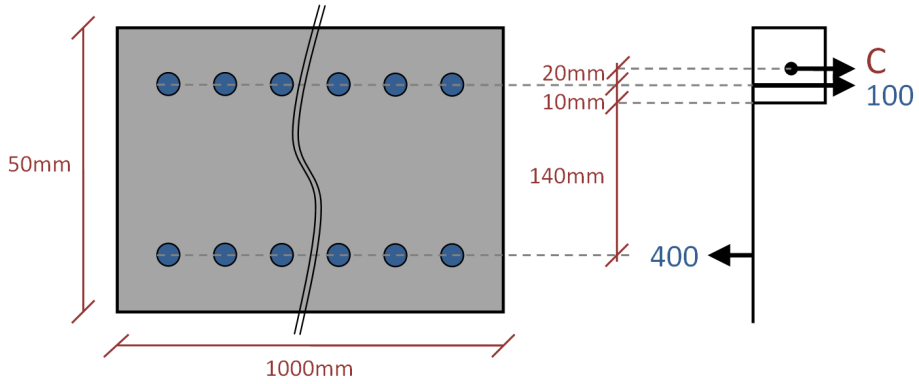


Figure F.6: Reinforced beam stress block - Case 3

Taking moments about centre of compression block (@ depth =60mm):

Moment capacity = (170 x 400) + (20 x 100) = 66000kNmm

Adequacy factor

AF = Moment capacity / Applied moment = 66000 / 1225 = 53.8776

LimitState:RING calculated adequacy factor

AF = 53.9 (to 3 significant figures)

Appendix G

Validation against bridge test results

G.1 Bolton laboratory tests (full-scale)

At Bolton Institute, UK, in the early 1990's a number of 3m and 5m span bridges were tested in the laboratory. Two of the bridges tested are shown on Figure G.1, Figure G.2, Figure G.3, and Figure G.4. A key advantage of these tests over those carried out in the field (e.g. see Section G.4) was that the internal constructional details and material properties were known.

LimitState:RING was originally developed to assist with the interpretation of the results from these laboratory tests. Since the original publication of the work in *The Structural Engineer* (Melbourne & Gilbert 1995, Melbourne et al. 1997, Gilbert & Melbourne 1994) the program has been significantly enhanced and for example now accounts for material crushing around hinges and includes more realistic models of the dispersion of the applied load through the backfill.

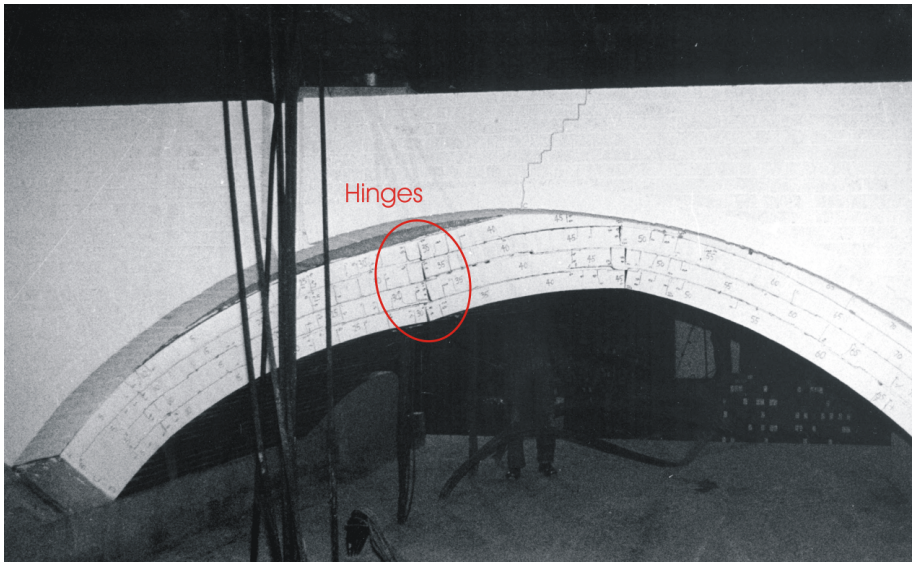


Figure G.1: Bolton bridge 5-2 containing debonded rings approaching collapse (note the diffused hinges under the load)

Figure G.2: Bolton bridge Multi-2 built with detached spandrel walls awaiting testing

Figure G.3: Bolton bridge Multi-2 approaching collapse (note that the left and centre spans are involved in the failure mechanism)

Figure G.4: Bolton bridge Multi-2 immediately following collapse (note that the right span has remained fully intact)

© *LimitState Ltd*

Bridge	Description	Experimental collapse load (kN)	LimitState:RING analysis (kN)		RING (B) / experiment
			(A) Default soil properties except using measured unit weight	(B) As (A) but using measured angle of soil friction (60°)	
3-1	3m single span	540	264	442	82%
5-1	5m single span	1720[a]	1178	1915	111%[a]
5-2	5m single span; debonded arch rings	500	253	400	80%
Multi-2	3m triple span	320	202	320	100%

[a] The experimental collapse load of this bridge was reduced by the sudden onset of partial ring separation

Table G.1: Sample comparison between Bolton laboratory and LimitState:RING collapse loads

Table G.1 presents LimitState:RING analysis results alongside experimental test results from Bolton Institute (representative bridges with detached spandrel walls are included since these behave demonstrably in a two dimensional manner). To obtain the LimitState:RING results, measured geometrical and unit weight properties were used together with a measured angle of friction of the soil backfill of 60°. (The backfill was purely frictional so zero cohesion was specified.) The computed failure load was found to be relatively insensitive to crushing strength so a value of 20N/mm^2 was used in all cases (experimentally recorded values for the brickwork used to construct the bridges ranged from 18.1 to 28.2N/mm^2).

It is clear from Table G.1 that predictions are quite conservative when the default soil angle of friction is used (column A), but become much more realistic when the measured value is used (column B[1]).

It should be noted that the over-prediction of the strength of bridge 5-1 results from the sudden onset of partial ring separation in the experimental load test. This is a quasi-brittle and unpredictable phenomenon[2] which cannot be modelled directly using ring, although the program can be used to try to bound the load carrying capacity from above (by modelling the barrel as a voussoir arch) and from below (by modelling the barrel as a series of separate arch rings).

The LimitState:RING data files corresponding to the aforementioned analyses are distributed with LimitState:RING (these are located in the Examples subdirectory, e.g:

C:\Program Files\LimitState\RING3.0\examples).

[1] Note that the predictions in column B differ from those given in the ring1.5 Theory and Modelling Guide principally because in the latter case a 45° cutoff angle for the Boussinesq load distribution model was specified, whereas the default value of 30° was used here.

[2] In fact a nominally identical bridge (bridge 5-3) tested subsequently failed at an even lower load of 1000kN, with failure again initiated by the onset of ring separation.

© LimitState Ltd

G.2 Sheffield laboratory tests (small-scale)

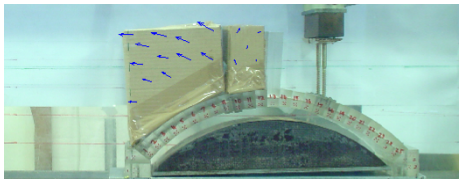
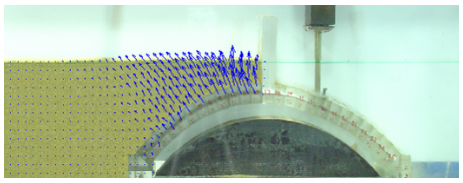
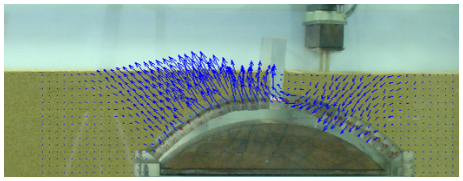
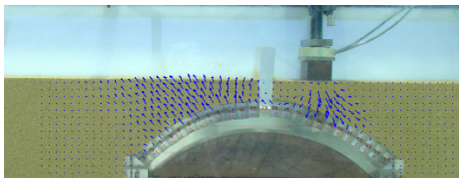
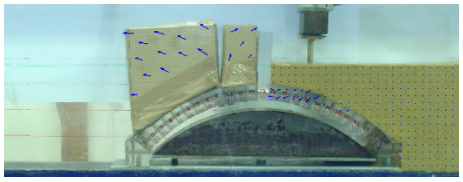
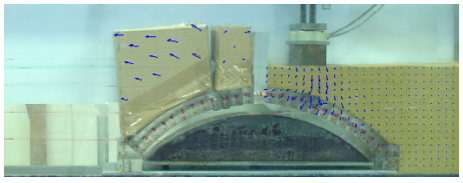
A series of small-scale tests were performed at the University of Sheffield to confirm the relative importance of passive restraint effects (i.e. as parts of the arch barrel remote from the load sway into the fill) and live load dispersion effects (i.e. as the live load spreads through the fill).

In these tests the applied load could be either applied to the surface of the fill or optionally directly onto the arch barrel. Also, to allow fill to optionally be placed only on one side, a keystone of extended height was used. Finally the fill on the passive side could optionally be contained either side of the three-quarter point hinge so as to act as a vertical dead load only. Further details of the tests are provided elsewhere (Gilbert et al. 2007).

Experimental and LimitState:RING results are summarised in Table G.2. In the LimitState:RING analyses measured geometrical and unit weight properties were used. The measured angle of soil friction was also used. Passive restraint, vertical dead weight over a half span and distribution of the load were switched off in line with the circumstances of the particular test arch being modelled.

It is evident from Table G.2 that the LimitState:RING predictions are remarkably good (all within 10% of the experimental results). This verifies that the simplified LimitState:RING soil model is capable of capturing the key effects of backfill.

© *LimitState Ltd*

Test [Key*]	**Photographs of model bridges with superimposed displacement vectors at peak load**	**Experimental peak load capacity (N)** [results without extended keystone]			**RING 3.0 analysis (N)**	**RING3.0 result / mean expt. result**
T1 [—]		107 [104]	108 [104]	107 [106]	99	93%
T2 [-P-]		141	142	140	133	94%
T3 [AP-]		138 [137]	137 [135]	137 [138]	132	96%
T4 [APL]		181 [178]	183 [177]	182 [179]	187	104%
T5 [A–]		103	104	100	97	95%
T6 [A-L]		130	131	136	136	103%

*A = Active; P = Passive; L = Load spreading

Table G.2: Results from load spread / passive restraint separation tests

G.3 Salford laboratory tests (full-scale)

To better establish the nature of the soil-arch interaction which takes place a series of bridges have recently been tested at the University of Salford, UK. The model bridges tested to date have been 3m in span and have been housed in a large, 8.3m long 2.1m high and extremely stiff test chamber incorporating large frictionless observation windows along one face to permit measurement of the soil and arch movements (Figure G.5 shows vectors of soil displacements

Bridge	Description	Experimental collapse load (kN)	LimitState:RING analysis (kN)		RING (B) / experiment
			(A) Default soil properties except using measured unit weight	(B) As (A) but also using measured soil properties	
1	3m single span - limestone fill	126	83.7	122	97%
2	3m single span - clay fill	92	95.5	95.5	104%

Table G.3: Sample comparison between Salford laboratory and LimitState:RING collapse loads

in the case of Bridge 1). Further details are available elsewhere (Gilbert et al. 2007).

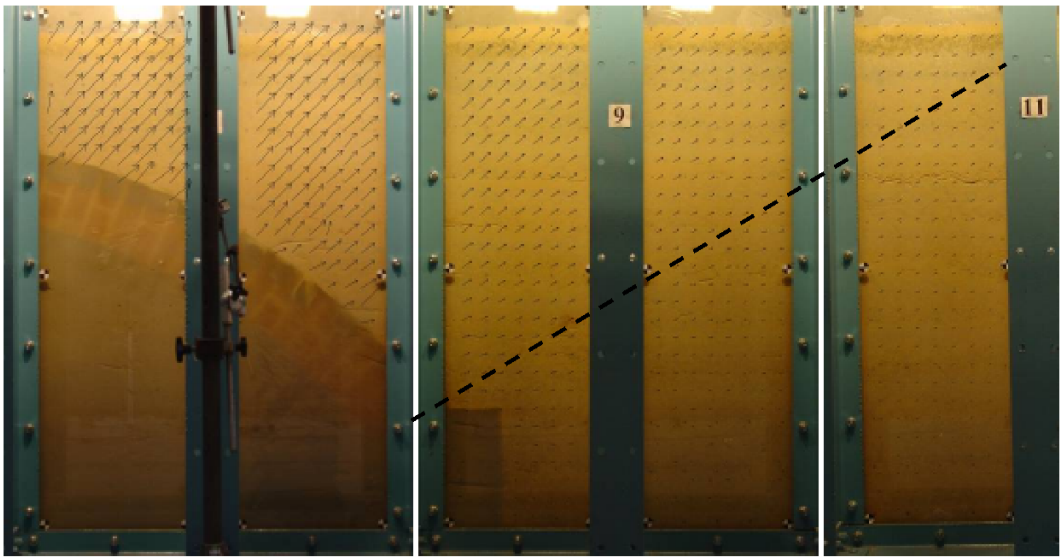


Figure G.5: Bridge 1: arch and backfill remote from the load, also showing soil displacement vectors

For the LimitState:RING analyses, measured geometry and unit weight properties were used initially (for sake of simplicity the soil unit weight for Bridge 2, which was clay filled with a limestone capping layer, was taken as the mean of the limestone and clay unit weights). For Bridge 1 the angle of friction of the soil was taken as 54.5° (the measured value). For Bridge 2, a cohesive strength of 78kPa was used (the measured value). Finally the masonry crushing strength was taken as 25MPa, which is representative of that found for the type of brickwork used. Experimental and analysis results are provided in Table G.3. It is clear from Table G.3 that, when the measured soil strength parameters are used, column (B), LimitState:RING provides a close prediction of load carrying capacity.

© *LimitState Ltd*

G.4 Field bridge tests

In the late 1980s and early 1990s, the Transport and Road Research Laboratory (TRRL, now TRL) in the UK carried out a series of load tests to collapse on redundant arch bridges. Most bridges failed in four hinge mechanisms, although some of the bridges were reported as failing by three hinge snap through or in compression (material failure). It was likely that many of the bridges tested were restrained considerably by their attached spandrel walls and/or masonry backing. Outline information on these bridge tests has been provided by Page (1993).

With the benefit of hindsight, significantly more pre- and post-test investigation work should have been performed to better characterise the internal construction details and material properties. This would have been useful in providing a more comprehensive data set for use by analysts who have since attempted to model the behaviour of the bridges under load.

In 2001 TRL were commissioned to independently validate 'ring' (the predecessor to LimitState:RING) and other available masonry arch bridge analysis software. Despite the uncertainties outlined above, as part of the validation process it was decided that the programs would be used to predict the carrying capacities of 5 of the field bridges load tested more than a decade previously. Details taken from the TRL report (Macfarlane & Ricketts 2001) relating to 'ring' are provided in Table G.4.

Bridge[a]	**Theoretical / experimental collapse load**
Torksey	81%
Bridgemill	100%
Barlae	92%
Preston	90%

[a] Strathmashie bridge was also modelled but was in poor condition and, because 'none of [the] defects were modelled during the analysis, all the programs returned non-conservative results'.

Table G.4: Correlation between TRL field bridge test and ring collapse loads (independently produced by TRL)

It is evident that agreement between the ring predictions and the full-scale test results was found to be reasonably good. The TRL report concluded that RING:

> "gives good results"

and

> "with some investment in an improved solver, [ring] would be a very effective tool for most assessment engineers"

The concern about the speed of the solver was addressed following the release of newer versions of RING which were up to 200x faster than RING 1.1 which was used in the 2001 TRL report.

G.5 Validation of the reinforcement model

A variety of checks have been undertaken to verify output from LimitState:RING in respect to the reinforcement model :

- To demonstrate that the software provides the same solutions as would be calculated by hand, a number of simple reinforced beam worked examples are included in Appendix F.

- The software has also been applied to a number of reinforced arch problems. Unfortunately much of the data available in the literature appears to be incomplete, or coloured by indeterminate factors, making accurate correlation difficult. Nevertheless, details of the validation work which has been undertaken are provided below.

G.5.1 Bradford arches

Chen (2004) and Chen et al. (2007) describe details of four tests performed on 2m span arch ribs (two reinforced and two unreinforced). Full details of the arches and reinforcement were available and these were used in LimitState:RING to predict load carrying capacities (for simplicity in all cases the masonry crushing strength was taken as 4.2N/mm^2, the value measured in the case of two of the four tests).

	Test load (kN)	**LimitState:RING prediction (kN)**
Unreinforced arches (x2)	1.4, 4.0	1.83
Reinforced arches (x2)	15.4, 18.4	21.1^A, 13.5^B

Table G.5: Validation of LimitState:RING results against Bradford 2m arch tests (after Chen 2004).

It is evident from Table G.5 that the results of the arch tests were quite variable (despite the two unreinforced and reinforced arches supposedly being identical). It is also clear that when full continuity of reinforcement at the supports is assumed in the analysis, the LimitState:RING predictions are non-conservative. However it appears that the structure supporting these arches may have been overly-flexible, potentially colouring the test results (e.g. no hinge crack was ever identified at the springing remote from the load in the case of one of the two reinforced arches). For this reason a further analysis was performed assuming no continuity of reinforcement at the supports; this successfully bounded from below the two reinforced arch test loads.

(Details of other Bradford arch bridges are available but these used mortar bonded multi-ring brickwork arch barrels, which typically failed abruptly due to the onset of partial ring separation (a brittle and highly unpredictable phenomenon). This makes them unsuitable for the present verification study.)

G.5.2 TRL laboratory bridge tests

A number of reinforced brickwork arch bridges have been constructed and tested in the laboratory by TRL (Sumon 2005). The first 5m span benchmark bridge comprised 3 debonded brickwork rings and granular soil fill material. The bridge failed at a load of 200kN.

Unfortunately some essential details of this bridge were either not made available, or are uncertain. For example angle of friction of the soil fill material is quoted at 40°, from 'manufacturer literature, but in fact this value will depend on the site compaction used. Also the strength of this bridge is likely to have been increased by test chamber side-wall friction, and by the confined nature of the arch fill. For these reasons the values of two of the parameters controlling soil-arch interaction were adjusted in order to give better agreement between the predicted and experimental results (Table G.6). The remaining input parameters were taken from Sumon (2005) or Chen (2004). (This includes the inter-ring friction value of 1.0 quoted by Chen, the origin of which is unclear and which appears rather high.)

Parameter	Value	Comments
Fill angle of friction	60°	Measured value in Bolton arch bridge tests
Max angle of dispersion of live loads	45°	

Table G.6: Soil-arch interaction parameter values used in TRL tests (others taken from Chen 2004, Sumon 2005).

The bridge was subsequently retro-reinforced using the MARS proprietary system, which involves inserting bars in slots formed in the intrados. The reinforcement comprised 622mm^2 steel positioned 19mm from the surface and with a yield strength of 480N/mm^2 (Yi, 2004). The bridge was then re-tested and to failure, reaching an estimated failure load of 320kN (a failure load of 276kN is also quoted by Sumon (2005), this apparently being the load at which displacement gauges were removed).

The bridge was analysed both before and after retrofitting. The values of all parameters used in the two analyses were identical, except that a steel rebar force of 149.3kN per metre width was entered for the retrofitted analysis (149.3 = 622 x 480 / 2000). It is evident from Table G.7 that the enhancement to the arch strength provided by the steel appears to be predicted well by LimitState:RING. The predicted failure mechanism of the reinforced arch is shown in Figure G.6.

(A number of other reinforced arch bridges were tested by TRL, but these contained radial pins, which is beyond the scope of this study.)

	Test load (kN)	LimitState:RING prediction (kN)
Arch 1 - Unreinforced benchmark	200	198
Arch 1 - Post retro-reinforcement	320	325

Table G.7: Validation of LimitState:RING results against TRL arch tests (after Chen 2004, Sumon 2005).

© LimitState Ltd

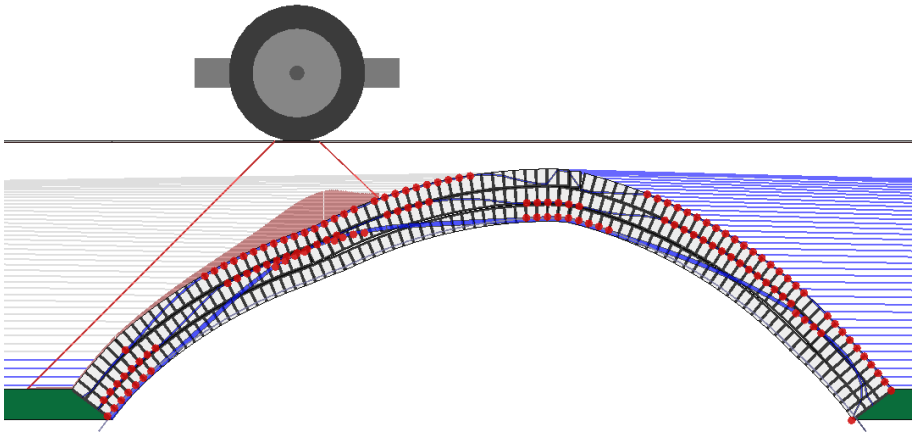


Figure G.6: Predicted LimitState:RING failure mechanism of retro-reinforced TRL Arch bridge

© *LimitState Ltd*

Appendix H

Comparison with previous versions

H.1 Version history

Table H.1 shows the various versions of ring which have existed since its original inception in 1992.

Version (Operating system)	Release year	Publicly available?	Comments
RING (DOS)	1992	No	Original research version, programmed in BASIC. Output published in papers appearing in the Structural Engineer.
RING1.0 (Win32)	1999	No	Windows user-interface added to original DOS version (program converted to Visual Basic). Boussinesq distribution model, multiple axle loads & user-defined arch profile capabilities added.
RING1.1 (Win32)	2001	Yes	Backfill elements added.
RING1.5 (Win32)	2004	Yes	Limit analysis kernel re-programmed in C++ and linked to existing Visual Basic user-interface. More efficient solution of multi-ring arch problems.
LimitState:RING 2.0 (Win32)	2007	Yes	Entire program re-written from scratch (in C++). Numerous features added.
LimitState:RING (Win32)	2011	Yes	Arch profile entry updated to include new types, including interpolated spline fit. Solver upgraded. Improved user interface implemented. Reinforcement and moment / force diagrams made available.

Table H.1: LimitState:RING version history

H.2 Comparison of results between versions

As part of the validation process for LimitState:RING, solutions for a number of benchmark problems have been compared with those from LimitState:RING 2.0 and RING 1.5. Sample results are shown on Table H.2

The results from LimitState:RING versions 2.0 and 3.0 are ostensibly identical, and also mostly very similar to results from RING 1.5. Where the difference in results exceeds 0.5% the reason for this is indicated.

The input files, which are used as validation tests, can be obtained from the LimitState website: www.limitstate.com/files/ringInputFiles.zip.

© *LimitState Ltd*

	Solution				
Bridge file name	***v1.5***	**%diff**	***v2.0***	**%diff**	***v3.0***
2-ring.rng	52.65	0.00	52.6511	0.00	52.6511
2-ring_crushing.rng	20.789	0.57[1]	20.9065	0.00	20.9064
2-ring_crushing_2.rng	38.6	3.32[1]	39.8829	0.00	39.8829
2-ring_crushing_nobackfill.rng	29.9	-0.21	29.8369	0.00	29.8369
2-ring_left-abutment_nohorizontal-pressures_2.rng	43.65	-0.22	43.555	0.00	43.555
2-ring_no-horizontal-pressures.rng	43.56	-0.01	43.555	0.00	43.555
2-ring_twin-axle.rng	49.73	0.00	49.7288	0.00	49.7288
2-ring_twin-axle_1.rng	49.73	0.00	49.7288	0.00	49.7287
2-ring_uniform-pressures.rng	113.5	0.00	113.502	0.00	113.502
2-span.rng	58.07	0.00	58.072	0.00	58.072
2-span_2-blocks-per-ring.rng	151.15	0.00	151.149	0.00	151.149
2-span_2-ring.rng	97.21	0.00	97.2059	0.00	97.2059
2-span_2-ring_2.rng	46.44	0.00	46.4396	0.00	46.4396
2-span_2-ring_3.rng	46.44	0.00	46.4396	0.00	46.4396
2-span_2-ring_crushing_backing.rng	156	0.02	156.025	0.00	156.025
2-span_2-ring_user-profile_crushing.rng	128	1.01[2]	129.296	0.00	129.296
2-span_2-ring_user-profile_crushing_2.rng	109	0.16	109.179	0.00	109.179
2-span_backing.rng	131.42	-3.34[3]	127.03	0.00	127.03
2-span_single-axle.rng	131.69	-0.02	131.668	0.00	131.668
2-span_single-axle_no-abutments.rng	226.47	0.00	226.461	0.00	226.461
2-span_single-axle_no-abutments_1.rng	131.67	0.00	131.668	0.00	131.668
2-span_single-axle_no-abutments_2.rng	83.75	0.00	83.7521	0.00	83.7521
2-span_twin-axle.rng	3.46	-0.09	3.45703	0.00	3.45703
3-spans.rng	1.88	-0.26	1.87503	0.00	1.87503
4-ring.rng	40.35	0.00	40.3484	0.00	40.3484
midLoad.rng	9.58	0.00	9.57958	0.00	9.57958
simple.rng	192.71	0.00	192.711	0.00	192.711
simple_1.rng	311.45	0.00	311.447	0.00	311.447
simple_2.rng	403.79	0.00	403.786	0.00	403.786
simple_3.rng	98.92	0.00	98.9232	0.00	98.9232
simple_4.rng	79.08	0.00	79.0813	0.00	79.0813
simple_5.rng	56.46	-0.01	56.4559	0.00	56.4559
simple_6.rng	175.45	0.00	175.451	0.00	175.451
simple_7.rng	56.38	-0.01	56.3747	0.00	56.3747
simple_8.rng	56.38	-0.01	56.3747	0.00	56.3747
simple_9.rng	60.01	-0.01	60.0052	0.00	60.0052
simple_crushing.rng	324	0.06	324.188	0.00	324.188
simple_crushing_2.rng	313	0.04	313.136	0.00	313.136
simple_crushing_3.rng	74.5	-0.13	74.4034	0.00	74.4031
simple_crushing_4.rng	98.55	-0.02	98.5263	0.00	98.5263
simple_crushing_geometrylocked.rng	-	inf[4]	943.187	0.00	943.187
simple_low-friction.rng	37.44	0.01	37.444	0.00	37.444
simple_no-backfill.rng	1.17	0.15	1.17172	0.00	1.17172
simple_user-profile.rng	125.57	0.00	125.569	0.00	125.569
simple_user-profile_crushing.rng	111	-0.31	110.652	0.00	110.652
user-profile_twin-axle.rng	27.37	-0.02	27.3656	0.00	27.3656
user-profile_twin-axle_1.rng	27.37	-0.02	27.3656	0.00	27.3656
simple_1-abutment_dispersion_1	60.69	6.85[5]	64.8456	0.00	64.8456
simple_1-abutment_dispersion_2	295.9	7.27[6]	317.421	0.00	317.421

[1] Less conservative results from LimitState:RING 2.0 due to improved solver for problems involving multi -rings and crushing.

[2] Due to error in LimitState:RING 1.x Boussinesq distribution model when load dispersed onto a bridge containing spans with user-defined arch profiles.

[3] From LimitState:RING 2.0 the backing height is conservatively always measured above the lowest point of the top surface of a skewback.

[4] The algorithm used in LimitState:RING 1. x prevented prediction of the load carrying capacity of bridges found to be 'geometrically locked' with infinite crushing strength, even if finite crushing strength was specified by the user.

[5] From LimitState:RING 2.0 the Boussinesq distribution spreads to the full extent of the specified cutoff cone, even if this means that load is applied to blocks with no 'direct line of sight' to the surface load. In LimitState:RING 1.x dispersion stops once 'direct line of sight' is lost.

[6] From LimitState:RING 2.0 it is assumed that load dispersed beyond a free -standing abutment is lost. In LimitState:RING 1.x no load is assumed to be lost.

Table H.2: Benchmark problems: comparison of results using different versions of LimitState:RING

© *LimitState Ltd*

Bibliography

Burroughs, P., Hughes, T., Hee, S. & Davies, M. 2002 Passive pressure development in masonry arch bridges. *Proc. Inst. Civ. Eng. Structures and Buildings* **152**(4), 331–339.

Chen, Y. 2004 *Strengthening of masonry arch bridges with near-surface reinforcement*, PhD thesis, University of Bradford, Department of Civil Engineering.

Chen, Y., Ashour, A. & Garrity, S. 2007 Modified four-hinge mechanism analysis for masonry arches strengthened with near-surface reinforcement. *Engineering Structures* **29**(8), 1864–1871.

Choo, B., Coutie, M. & Gong, N. 1991 Finite-element analysis of masonry arch bridges using tapered elements. *Proc. Inst. Civ. Eng. Part 2* (91), 755–770.

Department of Transport 2001 *DMRB Volume 3 Section 4 Part 3 - BD 21/01 - The Assessment of Highway Bridges and Structures*. Department of Transport.

Drucker, D. C. 1954 Coulomb friction, plasticity, and limit loads. *Journ. Appl. Mec. Trans. ASME* **21**(4), 71–74.

Gilbert, M. 1997 Gross displacement mechanism analysis of masonry bridges and tunnels, *Proceedings of the 11th International Brick/Block masonry conference*, Shanghai, pp. 473–482.

Gilbert, M. & Melbourne, C. 1994 Rigid-block analysis of masonry structures. *The Structural Engineer* **72**, 356–360.

Gilbert, M., Smith, C., Wang, J., Callaway, P. & Melbourne, C. 2007 Small and large-scale experimental studies of soil-arch interaction in masonry bridges, *Proc. 5th International Arch Bridges Conference, Madeira*, pp. 381–388.

Heyman, J. 1982 *The masonry arch*. Ellis Horwood, Chichester, United Kingdom.

Hillerbourg, A. 1976 Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. *Cement and concrete research* **6**, 773–782.

Hulet, K., Smith, C. & Gilbert, M. 2006 Load carrying capacity of flooded masonry arch bridges. *Proc. Inst. Civ. Eng. Bridge Engineering* **159**(3), 97–103.

International Union of Railways 1994 *Leaflet 774-2R - Distribution of axle-loads on ballasted railway bridges*. UIC.

Livesley, R. K. A. 1978 Limit analysis of structures formed from rigid blocks. *International Journal for Numerical Methods in Engineering* **12**, 1853–1871.

Macfarlane, A. & Ricketts, N. 2001 Evaluation of existing software for the assessment of masonry arch bridges, *Technical report*, TRL report to Railtrack.

Melbourne, A., Gilbert, M. & Wagstaff, M. 1997 The collapse behaviour of multi-span brickwork arch bridges. *The Structural Engineer* **75**(17), 297–305.

Melbourne, C. & Gilbert, M. 1995 The behaviour of multiring brickwork arch bridges. *The Structural Engineer* **73**, 39–47.

Melbourne, C. & Hodgson, J. 1995 The behaviour of skewed brickwork arch bridges, *1st Int Conf on Arch Bridges, Bolton*, pp. 309–320.

Network Rail 2006 *NR/GN/CIV/025 The Structural Assessment of Underbridges, Issue 3*. Network Rail.

Page, J. 1993 *Masonry Arch Bridges*. HMSO, UK.

Sumon, S. 2005 Innovative retrofitted reinforcement techniques for masonry arch bridges. *Proceedings of the ICE - Bridge Engineering* **158**(BE3), 91–99.

© *LimitState Ltd*

LimitState Ltd

The Innovation Centre
217 Portobello
Sheffield
S1 4DP
United Kingdom

t: +44 (0) 114 224 2240
e: info@limitstate.com
w: limitstate.com